

Reference R06: Steps for Computing Left and Right Riemann Sums

Given a continuous function $f(x)$ and an interval $[a, b]$ and some integer $n \geq 1$, the goal is to compute

The Left Riemann Sum with n rectangles, denoted L_n

and

The Right Riemann Sum with n rectangles, denoted R_n

Step 1: Compute the Rectangle Width $w = \Delta x = \frac{b-a}{n}$

Step 2: Make a list of the x coordinates of the edges of all the rectangles. (Notice, there will be $n + 1$ numbers) Find the corresponding y coordinates on the graph of $f(x)$.

x	$y = f(x)$
$x_0 = a$	$y_0 = f(x_0)$
$x_1 = a + \Delta x$	$y_1 = f(x_1)$
$x_2 = a + 2\Delta x$	$y_2 = f(x_2)$
$x_3 = a + 3\Delta x$	$y_3 = f(x_3)$
\vdots	\vdots
$x_{n-1} = a + (n-1)\Delta x$	$y_{n-1} = f(x_{n-1})$
$x_n = a + n\Delta x = b$	$y_n = f(x_n)$

Step 3: Use the y values on the list and Δx to compute L_n and R_n

$$L_n = (f(x_0) + f(x_1) + f(x_2) + \cdots + f(x_{n-1})) \cdot \Delta x$$

$$R_n = (f(x_1) + f(x_2) + f(x_3) + \cdots + f(x_n)) \cdot \Delta x$$