Reference R07: The Method of Integration by Substitution

Remember that the *Chain Rule for Derivatives* is used for taking the *derivative* of *nested functions*:

Chain Rule for Derivatives: $\frac{d}{dx}\,outer\big(inner(x)\big) = outer'\big(inner(x)\big)\cdot inner'(x)$

The goal now is to find the *general antiderivative* of a function $f(x)$ that involves a *nested function*. That is, we wish to find the *indefinite integral* $\int f(x)\,dx$ where the integrand $f(x)$ involves a nested function. This is not always possible. But sometimes it is, using the *Substitution Method*.

The Substitution Method for finding the *indefinite integral* $F(x) = \int f(x)\,dx$ where the integrand $f(x)$ involves a *nested function*.

Step 1 Identify the inner function and call it u. Write the equation $inner(x) = u$ to introduce the single letter u to represent the inner function. Circle the equation.

Step 2 Build the equation $dx = \frac{1}{u'}du$. To do this, first find u', then use it to build equation $dx = \frac{1}{u'}du$. Circle the equation.

Step 3 Substitute, Cancel, Simplify. In steps (1) and (2) you have two circled equations. **Substitute** these into the integrand of your indefinite integral. **Cancel** as much as possible and **simplify** by using the *Constant Multiple Rule*. The result should be a new basic integral involving just the variable u. (See **Remarks about Step 3** below.)

Step 4 Integrate. Find the new indefinite integral by using the indefinite integral rules. The result should be a *function form* involving just the variable u and $+C$.

Step 5 Substitute Back. Substitute $u = inner(x)$ into your function from Step (4) The result will be a new function form involving just the variable x and $+C$. This is the $F(x)$ that we seek. Present the result clearly as $F(x) = BLAH$ and circle it.

Remarks about Step 3: The result of **Step 3** should be a new indefinite integral with an integrand that is a function involving the variable u. There are three important things to check at the end of **Step 3**:

- There should be no x in the new indefinite integral. It should involve only u.
- The new indefinite integral should *not* involve a *nested function*, and it should be a *basic integral* that can be integrated using our indefinite integral rules.
- If the above two items are not satisfied, then either you made a mistake, or the original integral might be one for which the Substitution Method cannot be used.