Handout: Induction

New Rule of Inference: The Principle of Induction

$P(a)$ is true For all integers $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true. $\therefore$ For all integers $n \geq a$, $P(n)$ is true.

Usage:

- The letter a represents some fixed integer.
- The letters k and n represent variables whose domain is the set of all integers greater than or equal to a.
- The symbol $P(n)$ represents a predicate whose domain is the set of all integers greater than or equal to a

This new rule of inference will be used to prove statements of the form

Statement S: For all integers $n \geq a$, $P(n)$ is true.

Strategy for using the Principle of Induction

Preliminary work:

- Identify the number playing the role of a. (Introduce it.)
- Identify the predicate $P(n)$. (Introduce it in a sentence.)
- Figure out what the expressions for $P(a), P(k), P(k+1)$ look like. (Write them down.)

Build a proof of Statement *S* using the following structure:

Proof of Statement *S*:

Basis Step: Prove that $P(a)$ is true.

(1) The proof will begin somehow.

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* A bunch of steps may be involved. Usually a computation.

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(xx) $P(a)$ is true.

Inductive Step: Prove that for all integers $k \geq a$, if $P(k)$ is true, then $P(k+1)$ is true.

(1) Suppose that k is an integer such that $k \geq a$ and that $P(k)$ is true.

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* a bunch of steps may be involved

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(xx) $P(k+1)$ is true. (some justification goes here.)

Conclusion: For all integers $n \geq a$, $P(n)$ is true. (by the principle of induction)

End of Proof of Statement *S*.

The Principle of Strong Mathematical Induction

$P(a), P(a+1), \dots, P(b)$ are all true
For all integers $k \geq b$, if $P(a), P(a+1), \dots, P(k)$ are all true, then $P(k+1)$ is true.
$\therefore$ For all integers $n \geq a$, $P(n)$ is true.

Usage:

- The letter a represents some fixed integer.
- The letter n is a variable whose domain is the set of all integers greater than or equal to a.
- The symbol $P(n)$ represents a predicate whose domain is the set of all integers greater than or equal to a
- The letter b represents some fixed integer, with $a \leq b$.
- The letter k is a variable whose domain is the set of all integers greater than or equal to b.

This rule of inference will be used to prove statements of the form

Statement S: For all integers $n \geq a$, $P(n)$ is true.

Strategy for using the Principle of Strong Mathematical Induction

Preliminary work:

- Identify the number playing the role of a. (Introduce it.)
- Identify the predicate $P(n)$. (Introduce it in a sentence.)
- Write down the expressions for $P(a), P(a+1), \dots, P(b)$, $P(k+1)$.

Build a proof of Statement *S* using the following structure:

Proof of Statement *S*:

Basis Step: Prove that $P(a), P(a+1), \dots, P(b)$ are all true.

Inductive Step: Prove that for all integers $k \geq b$, if $P(a), P(a+1), \dots, P(k)$ are all true, then $P(k+1)$ is true.

(1) Suppose that k is an integer such that $k \geq a$ and $P(a), P(a+1), \dots, P(k)$ are all true.
*
*   a bunch of steps may be involved
*
(xx) $P(k+1)$ is true. (some justification goes here.)

Conclusion: For all integers $n \geq a$, $P(n)$ is true. (by the principle of induction)

End of Proof of Statement *S*