Relations Worksheet 1: The Statements and their Negations

Each of the four properties of relations is a logical statement. Each may be true or false. If the statement of one of the properties is true for a certain relation, then we say that the given relation *has that property*. If the statement of one of the properties is false for a certain relation, then we say that the given relation *does not have that property*. If the statement of one of the properties is false, then the negation of that statement will be true. Therefore, it is important to understand how to find the negations of each of the four statements above. Determine the negations of each of the four statements and write them here:

Words	Meaning
$\mathcal{R}$ is reflexive:	$\forall a \in A(a\mathcal{R}a)$
$\mathcal{R}$ is not reflexive:	
$\mathcal{R}$ is symmetric:	$\forall a, b \in A(If\ a\mathcal{R}b\ T\ then\ b\mathcal{R}a)$
$\mathcal{R}$ is not symmetric:	
$\mathcal{R}$ is transitive:	$\forall a, b, c, \in A(If\ (a\mathcal{R}b\ and\ b\mathcal{R}c)\ then\ a\mathcal{R}c)$
$\mathcal{R}$ is not transitive:	
$\mathcal{R}$ is an equivalence relation:	$\mathcal{R}$ is Reflexive and Symmetric and Transitive
$\mathcal{R}$ is not an equivalence relation:	

Relations Worksheet 2: Examples

For each of the following relations, draw a cartesian plane and sketch the points that are elements of the relation. Then decide if the relation is reflexive, symmetric, transitive. If a relation has one of the properties, then write "yes" in the box. If a relation does not have one of the properties, write "no" in the box and give a counterexample that shows that the relation does not have the property. That is, if you say that a relation is not reflexive, then you need to give an example of an x that shows that the relation is not reflexive. If you say that a relation is not symmetric, then you need to give an example of an x and a y that show that the relation is not symmetric. If you say that a relation is not transitive, then you need to give an example of an x, y, z that shows that the relation is not transitive.

	Relation	Reflexive?	Symmetric ?	Transitive?
$\mathcal{R}_0$	$x\mathcal{R}_0 y$ means $x - y = 2$		No: Let $(x, y) = (8,6)$	
$\mathcal{R}_1$	$x\mathcal{R}_1 y$ means $x < y$			
$\mathcal{R}_2$	$x\mathcal{R}_2 y$ means $x^2 + y^2 = 1$			No: Let $(x, y, z) = (1,0,1)$
$\mathcal{R}_3$	$x\mathcal{R}_3 y$ means $xy \neq 0$	No: Let $x = 0$		
$\mathcal{R}_4$	$x\mathcal{R}_4 y$ means $(y - x)(y - 2x) = 0$			
$\mathcal{R}_5$	$x\mathcal{R}_5 y$ means $x \leq y$			
$\mathcal{R}_6$	$x\mathcal{R}_6 y$ means $(y - x)xy = 0$			
$\mathcal{R}_7$	$x\mathcal{R}_7 y$ means $x^2 = y^2$			

Big hints

- for $\mathcal{R}_4$, $(y - x)(y - 2x) = 0$ is logically equivalent to $(y - x = 0)$ *or* $(y - 2x = 0)$.
- for $\mathcal{R}_6$, $(y - x)xy = 0$ is logically equivalent to $(y - x = 0)$ *or* $(x = 0)$ *or* $(y = 0)$.