Continuing Section 2.1 Rates of Change and Derivatives

Definition of the Tangent Line

Words: The line tangent to graph of f at a.

Usage: f is a function and a is a real number

Meaning: The line that has these two properties

(i) The line contains the point $(a, f(a))$
(so the line touches the graph of ~~f(a)~~ f at that point)
which is called the point of tangency

(ii) The line has slope

$$m = f'(a)$$

This is called the slope of the tangent line

Remark: Provided that $f(a)$ and $f'(a)$ both exist.

What about the equation of the tangent line?

Review: Suppose it is known that some line L has

known point (a, b)

known slope m

The easiest way to build an equation for line L is the point slope form of the equation of the line

$$(y-b) = m(x-a)$$

From here, we could convert to slope intercept form

$$y - b = m(x-a) = mx - ma$$

$$y = mx - ma + b$$

$$y = \underset{\text{slope}}{m}x + \underbrace{(b - ma)}_{y \text{ intercept}}$$

slope intercept form

Apply this to the tangent line

- known point $(a, f(a))$
- known slope $f'(a)$

Point slope form of equation of tangent line

$$y - f(a) = f'(a)(x-a)$$

a = x coordinate of point of tangency

$f(a)$ = y coordinate of point of tangency

$f'(a)$ = slope of tangent line

<u>Return to function from Wednesday's examples</u>

$$f(t) = 8t - t^2 = t(8-t)$$

(a) Find <u>slope</u> of line tangent to graph of $f(t)$ at $t=1$.

<u>Solution</u> We are being asked to find $m = f'(1)$

On Wednesday, we found

The instantaneous velocity at $t=1$ and the answer was $m = \lim\limits_{h \to 0} \frac{f(1+h) - f(1)}{h}$

$$\vdots$$

$$= 6$$

Another symbol for this quantity is $f'(1)$

So the tangent line slope is $m = f'(1) = 6$

(b) Find the equation of the line tangent to graph of f at $t=1$

(and convert to slope intercept form)

Solution

We need to build this equation

$$(y - f(a)) = f'(a)(x-a)$$

Strategy Get Parts

$a = 1$ (t coordinate of point of tangency)

$f(a) = f(1) = 8(1) - (1)^2 = 8 - 1 = 7$ (y coord of point of tangency)

$f'(a) = f'(1) = 6$ (slope of tangent line)

<u>Strategy Part 2</u> Put parts in equation and convert to slope intercept form

$$y - f(a) = f'(a)(t - a)$$

$$y - 7 = 6(t - 1)$$

Convert

$$y - 7 = 6t - 6$$

$$y = 6t + 1$$

Example 2 Let $f(x) = \sqrt{x}$

Find equation of line tangent to graph at $x = 9$

Solution

We need to build $y - f(a) = f'(a)(x-a)$

Get parts

$a = 9$ x coord point of tangency

$f(a) = f(9) = \sqrt{9} = 3$ y coord of point of tangency

need $f'(a) = f'(9)$

$$m = f'(9) = \lim_{h \to 0} \frac{f(9+h) - f(9)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$= \lim_{h \to 0} \frac{(\sqrt{9+h} - 3)}{h} \frac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$$

$$\vdots$$

$$= \lim_{h \to 0} \frac{(\cancel{9}+h) - \cancel{9}}{h(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{9+h} + 3)}$$

indeterminate

Since $h \to 0$, we know $h \neq 0$ so we can cancel $\frac{h}{h}$

$$= \lim_{h \to 0} \frac{1}{\sqrt{9+h} + 3}$$

$$= \frac{1}{\sqrt{9+0} + 3} = \frac{1}{3+3} = \frac{1}{6}$$

Part 2 Sub parts into equation

$$y - f(a) = f'(a)(x - a)$$

$$y - 3 = \frac{1}{6}(x - 9)$$

Solve for y

$$y - 3 = \frac{1}{6}x - \frac{9}{6} = \frac{1}{6}x - \frac{3}{2}$$

$$y = \frac{1}{6}x - \frac{3}{2} + 3$$

$$y = \frac{1}{6}x + \frac{3}{2}$$