Day 12 Math 2301 Section 110 (Barsamian) Mon Feb 6, 2023

- Pick up Graded Q2 (Solutions attached)
- Delfino's Office Hours: Tue/Thu 5:00-7:00 PM Morton 313
- Tomorrow: Recitation Exercises from Section 2.2
- Today Lecture } Section 2.2
- Wednesday Lecture } Section 2.2
- Friday: Exam X1 covers through section 2.2
  (front & back of two sheets of paper)

Recall from Friday + Wednesday

Definition of Instantaneous Rate of Change of f at a

words Instantaneous Rate of Change of f at a

words The derivative of f at a

words The instantaneous velocity at time a (in the case where the variable represents time and f is a position function)

symbol $f'(a)$

meaning the number m obtained by this limit calculation

$$\underbrace{m = \lim_{h \to 0} \underbrace{\frac{f(a+h) - f(a)}{h}}_{\substack{\text{average rate of change} \\ \text{slope of a secant line}}}}_{\substack{\text{instantaneous rate of change} \\ \text{slope of a tangent line}}}$$

graphical significance m is the slope of the line tangent to graph of f at $x=a$

Definition of Tangent line

words: the line tangent to graph of f at $x=a$

Meaning: the line that has these two properties

- The line contains the point $(a, f(a))$ the point of tangency
- The line has slope $m = f'(a)$ tangent line slope

Recall Friday's Example

$$f(x) = 8x - x^2$$

Find equation of line tangent to graph of f at $x=1$

Solution strategy build $(y - f(a)) = f'(a)(x-a)$

general equation for tangent line

results from Friday

$$a = 1$$

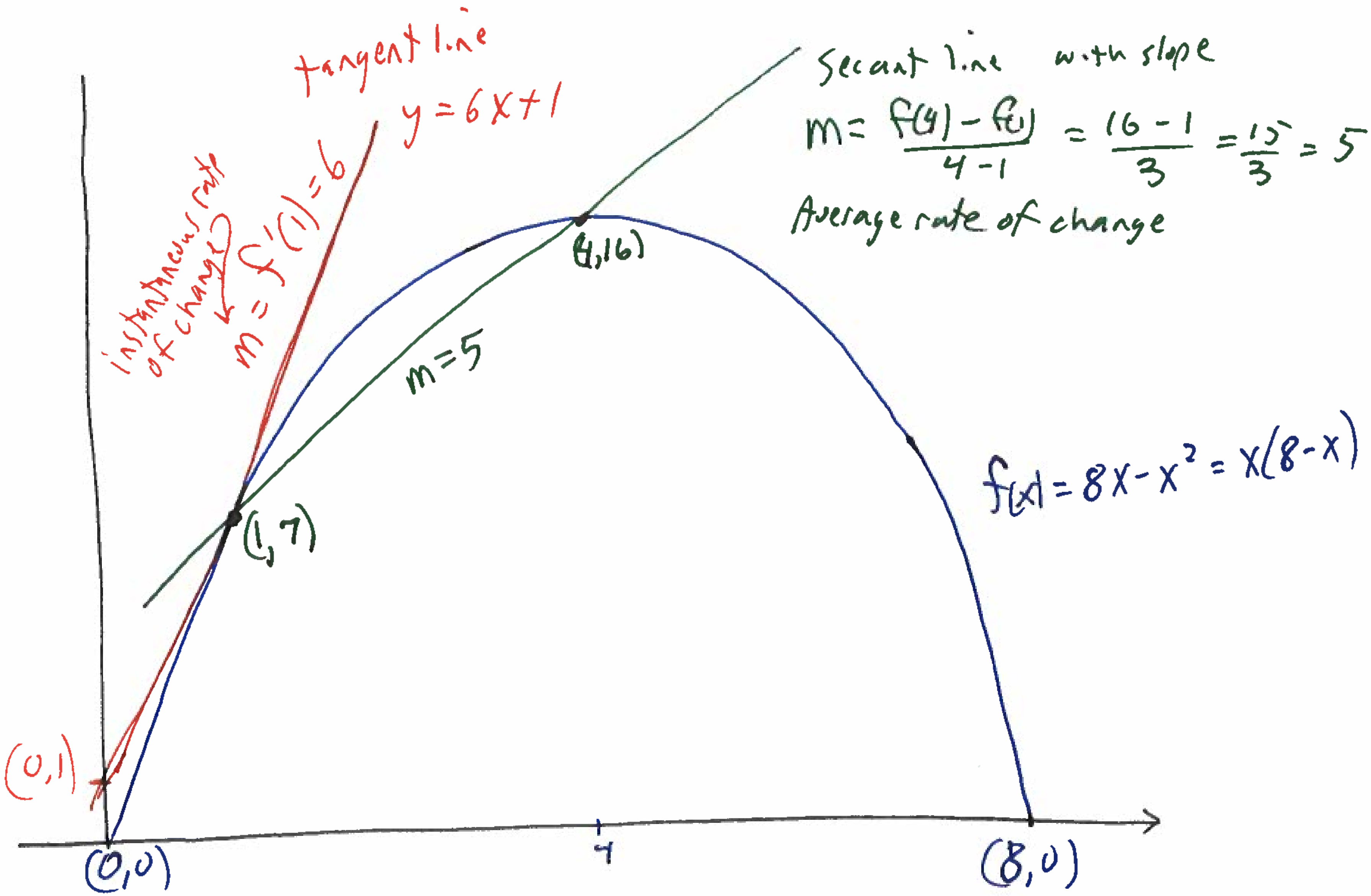
$$f(a) = f(1) = \dots = 7$$

$$f'(a) = f'(1) = \dots = 6$$

$$(y-7) = 6(x-1)$$

$$y = 6x+1$$

Now consider graph of f(x) and the tangent line



Section 2.2 The Derivative as a function

Consider this process

$f(x) = 8x - x^2$

input

a function

find the slope of the line tangent to the graph of f at 1

compute

$m = \lim_{h \to 0} \dfrac{f(1+h) - f(1)}{h}$

output

a number that is a slope

$m = 6 = f'(1)$

Now consider this process

$f(x) = 8x - x^2$

$f(x) =$ unknown

input a function

find the slope of the line tangent to graph of f at the number a

compute

$m = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h}$

output a number

a number

$m = 8 - 2a = f'(a)$

$m = \lim_{h\to 0} \dfrac{f(a+h) - f(a)}{h} = f'(a)$

a number

Finally consider this process

$f(x)$

input

function

the derivative machine

Compute

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

where x is a variable

output

a function

expression involving the variable x

$$f'(x) = \overbrace{\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

Definition of the Derivative

words: the derivative of f

symbol: $f'(x)$

meaning: The function obtained by computing this limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example #1

Let $f(x) = \sqrt{36 - x}$

ⓐ find domain of $f(x)$:

Can't take square root of negative number

So must have $36 - x \geq 0$

Add x to both sides

$36 \geq x$

$x \leq 36$

domain $(-\infty, 36]$

(b) Find $f'(x)$ using the Definition of the derivative

Solution we need to find $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

details

$$f(x) = \sqrt{36 - x}$$

$$f(\ ) = \sqrt{36 - (\ )}$$ empty version

$$f(x+h) = \sqrt{36 - (x+h)}$$

indeterminate form

$$= \lim_{h \to 0} \frac{\sqrt{36 - x - h} - \sqrt{36 - x}}{h} \cdot \frac{(\sqrt{36 - x - h} + \sqrt{36 - x})}{\sqrt{36 - x - h} + \sqrt{36 - x}}$$

$$\vdots$$

$$= \lim_{h \to 0} \frac{(\cancel{36} \cancel{-x} - h) - (\cancel{36} \cancel{-x})}{h(\sqrt{36 - x - h} + \sqrt{36 - x})}$$

$$= \lim_{h \to 0} \frac{-h}{h(\sqrt{36 - x - h} + \sqrt{36 - x})}$$ Still indeterminate

Since $h \to 0$, we know $h \neq 0$, so we can cancel $\frac{h}{h}$

$$= \lim_{h \to 0} \frac{-1}{\left(\sqrt{36-x-h} + \sqrt{36-x}\right)}$$

no longer indeterminate

$$= \frac{-1}{\sqrt{36-x-(0)} + \sqrt{36-x}}$$

$$= \frac{-1}{\sqrt{36-x} + \sqrt{36-x}}$$

$$f'(x) = -\frac{1}{2\sqrt{36-x}}$$