MATH 2301 Section 110 (Barsamian) Day 16 (Mon Feb 13) (1)

Please sit in pairs

Last week Section 2.2 we found the derivative $f'(x)$ using the Definition of the Derivative

$$f'(x) = \lim_{h \to 0} \underbrace{\frac{f(x+h) - f(x)}{h}}_{\text{you had to build this limit and evaluate it.}}$$

Hard!

S.I. Tue 4:45-5:45 Thurs 5:15-6:15 in Morton 227

Meeting #16 (Mon Feb 13)

~~3~~ 2

# This week: Section 2.3 Basic Differentiation Properties

Rules for certain derivatives that occur frequently

~~The Concept~~

## Notation for derivatives

Symbol: $f'(x)$ or $y'$

Symbol: $\frac{d}{dx} f(x)$ or $\frac{d}{dx} y$

Symbol: $\frac{df(x)}{dx}$ or $\frac{dy}{dx}$

Spoken: dee f(x) dee x

or dee dee x of f(x)

dee y dee x

or dee ~~dee x~~ dee x of y

## First Derivative Rule

### The Constant Function Rule

Two equation version

If $f(x) = c$ then $f'(x) = 0$

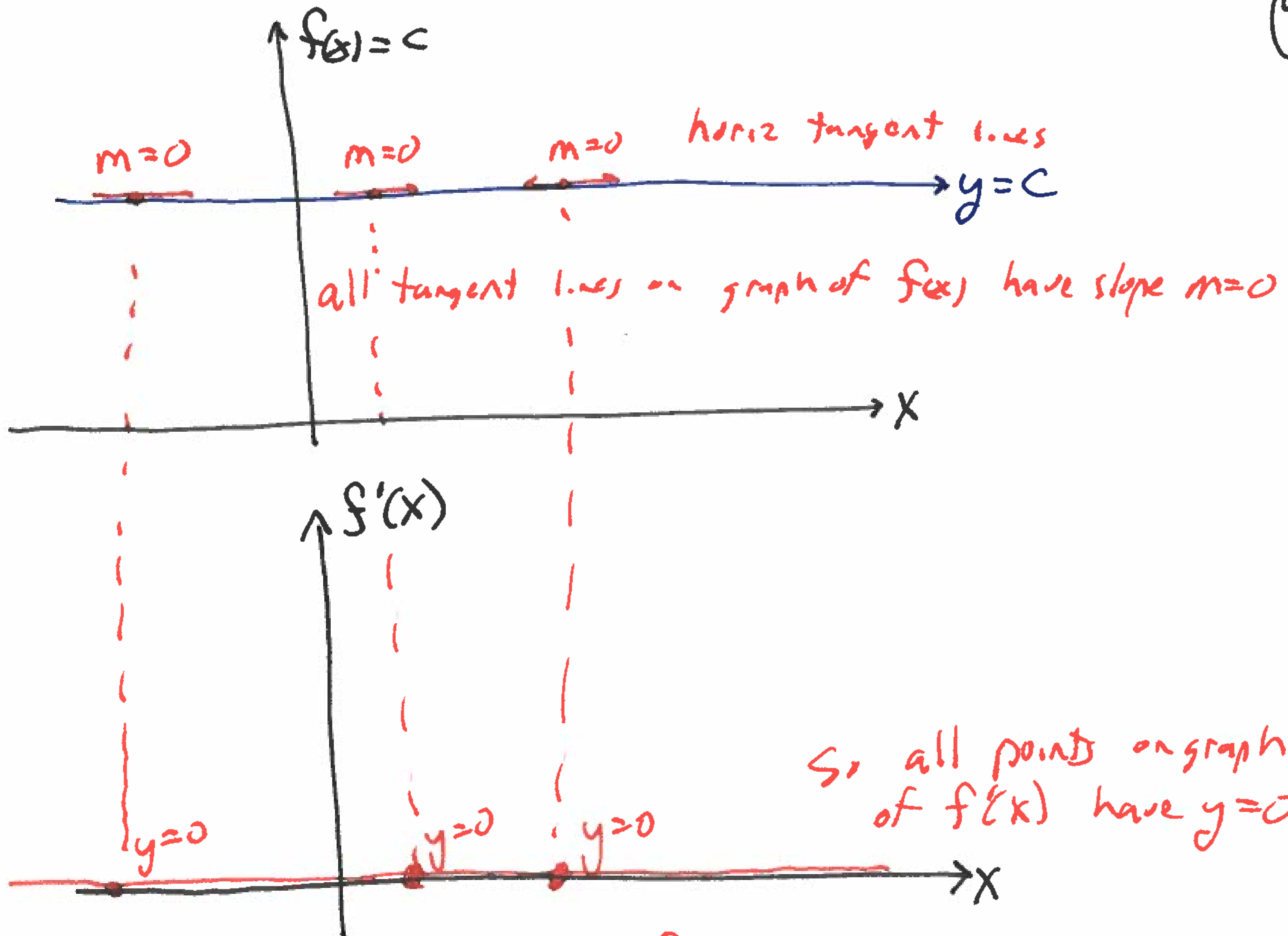
constant

Single equation version

$$\frac{d}{dx} c = 0$$

This is proven (using definition of derivative) on p. 95

Does this make sense? Consider Graphs




So $f'(x) = 0$

## Another Derivative Rule

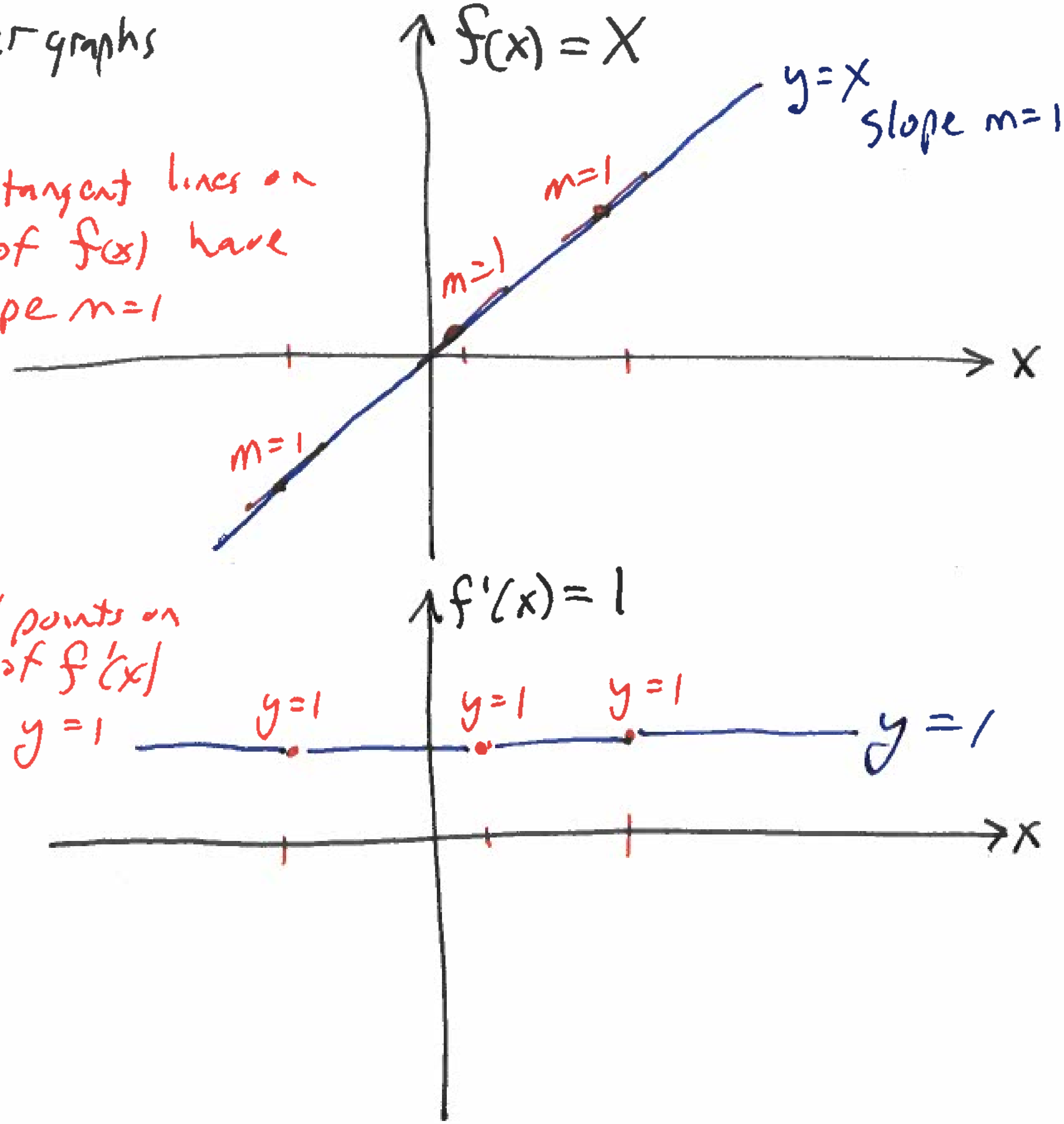
$$\boxed{\begin{array}{l} \underline{\text{Two equation version}} \\ \text{If } f(x) = x \quad \text{then } f'(x) = 1 \\ \underline{\text{Single equation version}} \\ \qquad \frac{d}{dx} x = 1 \end{array}}$$

this could be proven using <u>definition of derivative</u>

# Does this rule make sense? Is it believable?

Consider graphs

all tangent lines on graph of $f(x)$ have slope $m=1$




So all points on graph of $f'(x)$ have $y=1$

## The power rule

Used for finding the derivative of a power function

That is, function of the form $f(x) = x^P$

real number exponent

variable base

### The Power Rule

Two equation version

$$\text{If } f(x) = x^P \quad \text{then} \quad f'(x) = P \cdot x^{P-1}$$

Single equation version

$$\frac{d}{dx} x^P = P \cdot x^{P-1}$$

## Example

Let $f(x) = x^5$

Find $f'(x)$

## Solution

$$f'(x) = \frac{d}{dx} x^{\overset{\leftarrow p=5}{5}} \underset{\substack{\uparrow \\ \text{power rule} \\ \text{with } p=5}}{=} 5 \cdot x^{5-1} = 5x^4$$

## Bad Solution

$$\cancel{\frac{d}{dx} x^{5-1} = 5x^4}$$

not actually true

Example

Let $g(x) = \dfrac{1}{x^5}$

Find $g'(x)$

~~Rea~~ Solution  Must rewrite $g(x)$ in power function form

$$g(x) = \frac{1}{x^5} = x^{-5}$$

power function form

Then take derivative

$$g'(x) = \frac{d}{dx} x^{-5} \underset{\substack{\uparrow \\ \text{power Rule} \\ \text{with } p=-5}}{=} (-5)x^{-5-1} = -5x^{-6} = -5 \cdot \frac{1}{x^6} = \frac{-5}{x^6}$$

p = -5

power function form

convert to positive exponent form

Observe $\quad \dfrac{d}{dx}\dfrac{1}{x^5} \neq \dfrac{1}{\frac{d}{dx}x^5}$

Be careful about notation

Bad solution

$$\frac{d}{dx}\frac{1}{x^5} = x^{-5} = -5x^{-6} = -5\frac{1}{x^6} = \frac{-5}{x^6}$$

not true   not true

To make this true, we would need to add some symbols

$$\frac{d}{dx}\frac{1}{x^5} = \frac{d}{dx}x^{-5} = -5x^{-6} = -5\frac{1}{x^6} = \frac{-5}{x^6}$$

Calculations need to have a valid left side

# The Sum Rule and Constant Multiple Rule

$$\boxed{\frac{d}{dx}\left(a f(x) + b g(x)\right) = a \frac{d}{dx} f(x) + b \frac{d}{dx} g(x)}$$

a,b constants

f,g functions

**Example** Find derivative of $f(x) = 1.4x^5 - 2.5x^2 + 6.7$

**Solution**

$$f'(x) = \frac{d}{dx}\left(1.4x^5 - 2.5x^2 + 6.7\right)$$

constant function

$\leftarrow p=5$ $\quad$ $\leftarrow p=2$

$$= 1.4\frac{d}{dx}x^5 - 2.5\frac{d}{dx}x^2 + \frac{d}{dx}6.7$$

by Sum & Constant multiple rule

$$= 1.4\left(5x^{5-1}\right) - 2.5\left(2x^{2-1}\right) + (0)$$

By Power Rule and constant function Rule

$$= 7x^4 - 5x$$

## Class Drill: Rewriting $f(x)$ in *Power Function Form*, then Differentiating (Section 2.3)

### Part 1: Rewriting Functions in Different Forms

Fill in the empty spaces in this table.

| Simplified Form Involving Radicals and/or Positive Exponents | = | Positive Exponent Form with Separate Constants | = | Power Function Form (Sum of terms of form *constant × power function*) $ax^p + bx^q$ |
|---|---|---|---|---|
| $f(x) = \frac{5}{x^2} + \frac{9}{x}$ | = | $5\left(\frac{1}{x^2}\right) + 9\left(\frac{1}{x}\right)$ | = | $5x^{-2} + 9x^{-1}$ |
| $f(x) = \frac{1.2}{\sqrt{x}} - \frac{0.6}{\sqrt[3]{x^2}}$ | = | $1.2\left(\frac{1}{x^{1/2}}\right) - 0.6\left(\frac{1}{x^{2/3}}\right)$ | = | $1.2x^{-1/2} - 0.6x^{-2/3}$ |
| $f(x) = \frac{5}{\sqrt[3]{x}} - \frac{6}{x^{1/2}}$ | = | $5\left(\frac{1}{x^{1/3}}\right) - 6\left(\frac{1}{x^{1/2}}\right)$ | = | $\boxed{5x^{-1/3} - 6x^{-1/2}}$ |
| $-\frac{10}{7x^3} + \frac{9}{13x^2}$ | = | $-\frac{10}{7}\left(\frac{1}{x^3}\right) + \frac{9}{13}\left(\frac{1}{x^2}\right)$ | = | $-\left(\frac{10}{7}\right)x^{-3} + \left(\frac{9}{13}\right)x^{-2}$ |
| $f(x) = \frac{7\sqrt[3]{x}}{5} + \frac{3}{11x^{2/5}}$ | = | $\frac{7}{5}\left(x^{1/3}\right) + \frac{3}{11}\left(\frac{1}{x^{2/5}}\right)$ | = | $\frac{7}{5}x^{1/3} + \frac{3}{11}x^{-2/5}$ |
| $\frac{7}{15x^{2/3}} - \frac{6}{55x^{2/5}}$ | = | $\frac{7}{15}\left(\frac{1}{x^{2/3}}\right) - \frac{6}{55}\left(\frac{1}{x^{2/5}}\right)$ | = | $\left(\frac{7}{15}\right)x^{-2/3} - \left(\frac{6}{55}\right)x^{-7/5}$ |

Part 2 is on back →

$\textcircled{HB}$ 15

## Part 2: Finding a Derivative Using Sum Rule, Constant Multiple Rule, Power Rule

$$f(x) = \frac{7\sqrt[3]{x}}{5} + \frac{3}{11x^{2/5}}$$

(A) Rewrite $f(x)$ in ***power function form***.
That is, rewrite it as a sum of terms of the form *constant* $\times$ *power function*. That is, $ax^p + bx^q$.
(**Hint:** You have already done this part on the previous page!)

$$f(x) = \frac{7\sqrt[3]{x}}{5} + \frac{3}{11x^{2/5}} = \cdots = \frac{7}{5}x^{1/3} + \frac{3}{11}x^{-2/5}$$

(B) Find $f'(x)$.

- Use the techniques of Section 2.5. (That is, DO NOT use the Definition of the Derivative.)
- Show all details clearly and use correct notation.
- Simplify your final answer, and rewrite it so that it does not have any negative exponents.
(**Hint:** You have already done the necessary simplifying on the previous page!)

$$f'(x) = \frac{d}{dx}\left(\frac{7}{5}x^{1/3} + \frac{3}{11}x^{-2/5}\right)$$

$\leftarrow p=\frac{1}{3}$ $\qquad \leftarrow p = -\frac{2}{5}$

$$= \frac{7}{5}\frac{d}{dx}x^{1/3} + \frac{3}{11}\frac{d}{dx}x^{-2/5}$$

$$= \frac{7}{5}\left(\frac{1}{3}x^{\frac{1}{3}-1}\right) + \frac{3}{11}\left(\left(-\frac{2}{5}\right)x^{-\frac{2}{5}-1}\right)$$

$$= \frac{7}{15}x^{-2/3} - \frac{6}{55}x^{-7/5}$$

$$\vdots$$

$$= \frac{7}{15x^{2/3}} - \frac{6}{55x^{7/5}}$$