Day 2 Wed Jan 18

Section 1.3 Limits

~~Q~~ Let $f(x) = \frac{x^2 - 2x - 3}{x - 3}$ $\qquad g(x) = x + 1$

Are these the same function?

Observe $f(x) = \frac{x^2-2x-3}{x-3} = \frac{(x+1)(x-3)}{(x-3)}$ $\qquad g(x) = x+1$

Are these the same function?

Check

x	$f(x) = \frac{(x+1)(x-3)}{(x-3)}$	$g(x) = x+1$
0	$f(0) = \frac{(0+1)(0-3)}{0-3} = \frac{1\cancel{(-3)}}{\cancel{(-3)}} = 1$	$g(0) = 0+1 = 1$
1	$f(1) = \frac{(1+1)(1-3)}{(1-3)} = \frac{2\cancel{(-2)}}{\cancel{(-2)}} = 2$	$g(1) = 1+1 = 2$
2	$f(2) = \frac{(2+1)(2-3)}{(2-3)} = \frac{3\cancel{(-1)}}{\cancel{(-1)}} = 3$	$g(2) = 2+1 = 3$
3	$f(3) = \frac{(3+1)(3-3)}{(3-3)} = \frac{4(0)}{(0)} = \frac{0}{0}$ does not exist!!	$g(3) = 3+1 = 4$

<u>Not</u> the same function

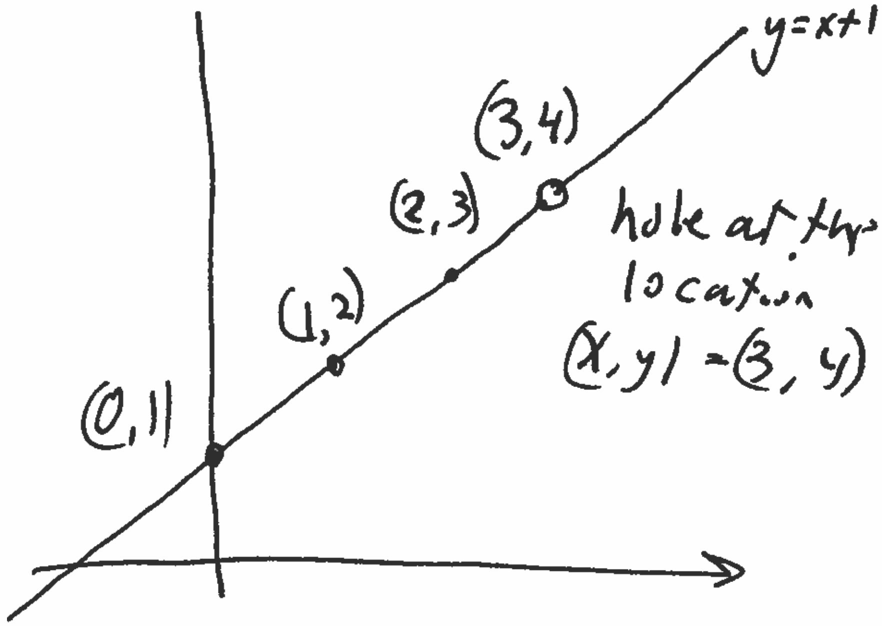
Different domains

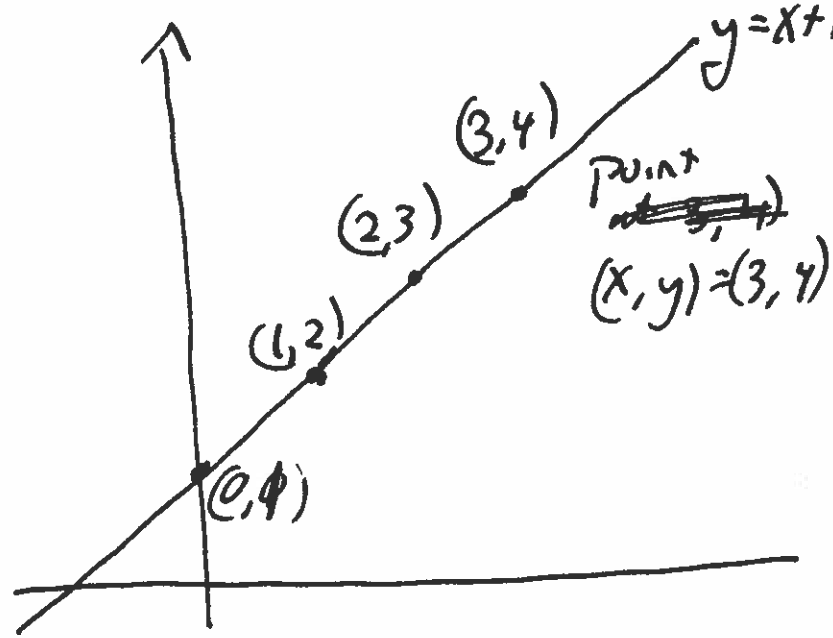
domain of f: all $x \neq 3$

domain of g: all x

graph $f(x) = \dfrac{x^2 - 2x - 3}{x - 3} = \dfrac{(x+1)(x-3)}{(x-3)}$

$g(x) = x + 1$





Another (related) question

When can you cancel $\frac{x}{x}$?

If $x=5$ then $\frac{x}{x} = \frac{\cancel{5}}{\cancel{5}} = 1$

If $x=-17$ then $\frac{x}{x} = \frac{\cancel{-17}}{\cancel{-17}} = 1$

But if $x=0$ then $\frac{x}{x}$ is $\frac{0}{0}$ which is undefined.

<u>Conclusion</u> If you don't know anything about the value of x, then you cannot cancel $\frac{x}{x}$

<u>However</u> if you have info that tells you that $x \neq 0$

then you can cancel!

$$\frac{X}{X} = 1$$

can cancel
because $X=0$

Most important concept of 1st Month of Calculus:

$\boxed{\text{When can you cancel terms, and } \underline{\text{why}}\text{?}}$

Limits (Section 1.3)

Definition of Limit

Symbol: $\lim_{x \to a} f(x) = L$

Spoken: The limit, as x approaches a, of f(x) is L.

usage: a is a real number
L is a real number
f is a function

informal meaning: When x gets closer & closer to "a" but not equal to "a", the values of f(x) get closer & close to L.

precise meaning: For every real number $\varepsilon > 0$ (epsilon), there exists a real number $\delta > 0$ (delta) such that If $x \neq a$ but $|x-a| < \delta$ then $|f(x) - L| < \varepsilon$

we won't be covering this

graphical significance: The graph appears to be heading for the location $(x,y) = (a, L)$

Example Consider $f(x) = \dfrac{x^2-2x-3}{x-3} = \dfrac{(x+1)(x-3)}{(x-3)}$

~~Consider $\lim_{x \to 3}$~~

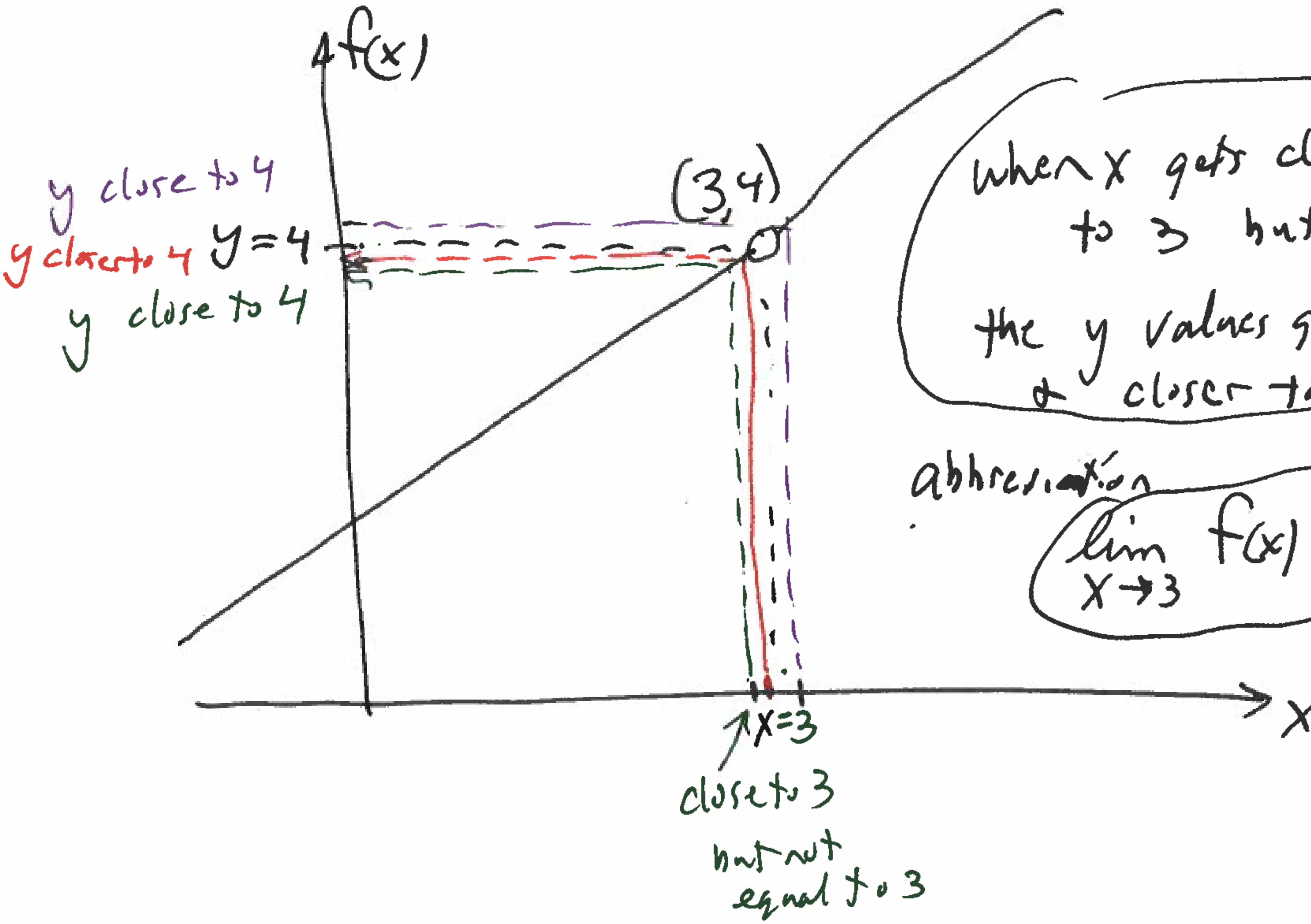
We know $f(3)$ does not exist.

What about $\lim_{x \to 3} f(x)$?

We will do this two ways.

Way 1 Graphical approach

graph of $f(x)$ $\longrightarrow$ describe limit behavior



when x gets closer & closer to 3 but not equal to 3

the y values get closer & closer to 4.

abbreviation

$$\lim_{x \to 3} f(x) = 4$$

Way 2 Estimate the limit by making a table of (x, y) values

can cancel non-zero terms

x	$f(x) = \frac{(x+1)(x-3)}{(x-3)}$
2.9	$f(2.9) = \frac{(2.9+1)\cancel{(2.9-3)}}{\cancel{(2.9-3)}} = 3.9$
2.99	$f(2.99) = \frac{(2.99+1)\cancel{(2.99-3)}}{\cancel{(2.99-3)}} = 3.99$
2.999	$f(2.999) = \frac{(2.999+1)\cancel{(2.999-3)}}{\cancel{(2.999-3)}} = 3.999$

x getting closer and closer to 3 from the left

y values getting closer & closer to 4

estimate $\lim_{x \to 3^-} f(x) = 4$

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Similar table

x	$f(x)$
3.1	$f(3.1) = \cdots = 4.1$
3.01	$f(3.01) = \cdots = 4.01$
3.001	$f(3.001) = \cdots = 4.001$

y values getting closer & closer to 4

x getting closer & closer to 3

from the right

$$\lim_{x \to 3^+} f(x) = 4$$

Conclude

Since left limit = right limit

we can write

$$\lim_{x \to 3} f(x) = 4$$