Today: Section 2.7 Related Rates

Wednesday we discussed 2.6 Implicit Differentiation

Start with
Equation Involving
x & y

process called
Implicit differentiation
involving taking $\frac{d}{dx}$
of both sides of the
equation
result is a new equation
involving $x, y, \frac{dy}{dx}$
Solve this equation for $\frac{dy}{dx}$

Result is
equation of the form
$\frac{dy}{dx}$ = stuff involving
x and y.

Remember the Terminology of Rate of Change

Given a function $f(t)$ t is the variable

words: The instantaneous rate of change of f at $t=a$

meaning: $f'(a)$

Remember: If $f(t)$ is a position function, describing the position of an object moving in 1-dimension, the quantity $f'(a)$ is called the instantaneous velocity at $t=a$

The Idea of Related Rates Problems

Start with an equation involving various quantities, some of which are actually functions of time t.

Process

Take $\frac{d}{dt}$ of both sides of the equation. The result will be a new equation involving the original quantities and $\frac{d}{dt}$ of those quantities. (the rates of change of the quantities)

The equation expresses a relationship between the rates of change

Hence, the name "related rates"

∗ can solve the equation for one of those rates of change

~~Method of Related Rates~~

Related Rates Problems:

<u>Given</u> various quantities that are related by an <u>equation</u>, and <u>given</u> values for certain of the <u>quantities</u> and <u>given</u> values for certain of their <u>rates of change</u>

<u>Goal</u>: Find some unknown rate of change.

Method of Related Rates

Step 1 Draw a picture of the situation

Step 2 Label picture with variables for important quantities.
(quantities that are implicated in the problem statement)

Step 3 Also add to the picture actual numbers for known quantities or known rates of change

Step 4 Identify the goal! Identify the unknown rate that you want to find.

Step 5 Figure out an equation that relates the quantities

Step 6 Use implicit differentiation to find $\frac{d}{dt}$ of both sides of this equation. result will be a new equation involving the various quantities and their rates of change.

Step 7 Solve this equation for the unknown rate

Step 8 Substitute in known values, simplify, and enjoy! :)

Example #1 2.7 #12

At noon, Ship A is 150 km west of Ship B.

Ship A is sailing East at 35 $\frac{km}{hour}$

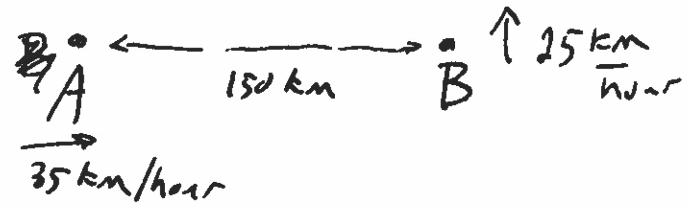
Ship B is sailing North at 25 $\frac{km}{hour}$

How fast is the distance between the ships changing at 4pm?

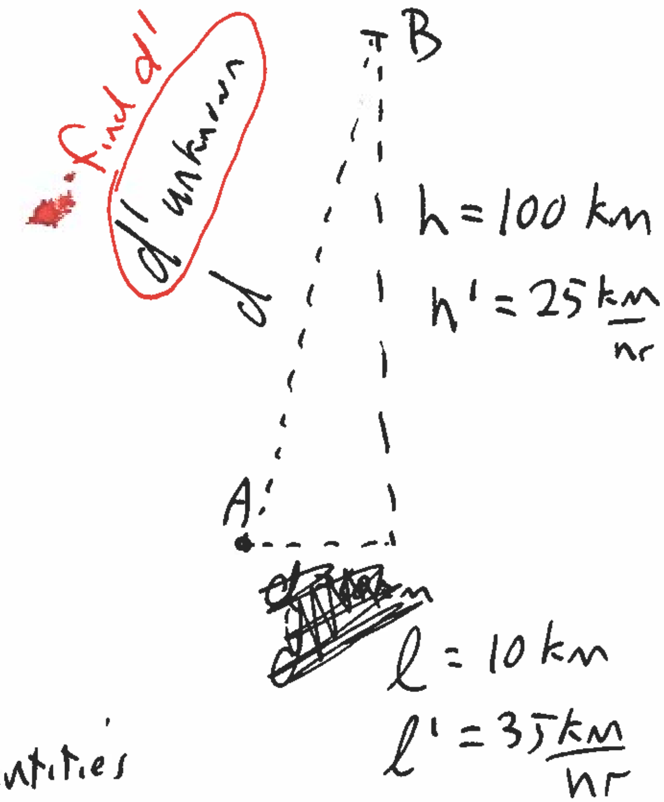
unknown rate

Step 1 Draw picture

Picture at noon



Picture at ~~noon~~ 4pm



Step 2 ✓ quantities: l, h, d

Step 3 ✓

Step 4 ✓ goal: find d'

Step 5 equation that relates the quantities

$$l^2 + h^2 = d^2$$ Pythagorean theorem

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Show time dependence

$$(l(t))^2 + (h(t))^2 = (d(t))^2$$

<u>Step 6</u> find $\frac{d}{dt}$ of both sides

$$\frac{d}{dt}\left((l(t))^2 + (h(t))^2 \right) = \frac{d}{dt}(d(t))^2$$

$$\underbrace{\frac{d}{dt}(l(t))^2}_{\text{chain rule}} + \underbrace{\frac{d}{dt}(h(t))^2}_{\text{chain rule}} = \underbrace{\frac{d}{dt}(d(t))^2}_{\text{chain rule}}$$

$$2 \cdot l(t) \cdot l'(t) + 2 \cdot h(t) \cdot h'(t) = 2 \cdot d(t) \cdot d'(t)$$

Simplify $l \cdot l' + h \cdot h' = d \cdot d'$

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Step 7 Solve for d'

$$d' = \frac{l \cdot l' + h \cdot h'}{d}$$

Step 8 Substitute in the known values

$$d' = \frac{10 \cdot 35 + 100 \cdot 25}{\sqrt{10^2 + 100^2}}$$

⋮

Simplify to get final answer

$$= \frac{350 + 2500}{\sqrt{100 + 10{,}000}} =$$

$$d' = \frac{2850}{\sqrt{10{,}100}}$$