MATH 2301 Section 110 (Barsamian) Day 27 (Fri March 3)

- Today: Q5
- Next Friday: X2

Section 3.1 Exponential functions

Introduce the number e.

behavior #1

- Suppose p is a real number. Then $1^p = 1$
- So $\lim_{p\to\infty} 1^p = 1$

behavior #2

- But if $b>1$, then b^p gets really big when $p\to\infty$.

  example $b=2$

  $2^1=2$ $\quad 2^2=4$ $\quad 2^3=8$ $\quad 2^4=16$ $\quad 2^5=32$

  So $\lim_{p\to\infty} 2^p = \infty$

  So for any base $b>1$, it will turn out that $\lim_{p\to\infty} b^p = \infty$

Consider this new limit

exponent is getting huge and positive

$$\lim_{x \to 0^+} \left((1 + x) \right)^{\frac{1}{x}}$$

base is getting closer and closer to 1.
(but is always slightly greater than 1)

It would be believable if this limit turned out to be 1 because of behavior #1.

But it would also be believable if that limit turned out to be ∞ because of behavior #2

But it would also be believable if the limit turned out to be some other number explore

x	$(1+x)^{1/x}$
1	$2 = (1+1)^{1/1}$
0.1	$(1+0.1)^{\frac{1}{0.1}} = (1.1)^{10} \approx 2.5937425$
0.01	≈ 2.7048138
0.001	≈ 2.7169239
0.0001	2.7181459
0.00001	2.7182682

Big Facts from Higher math

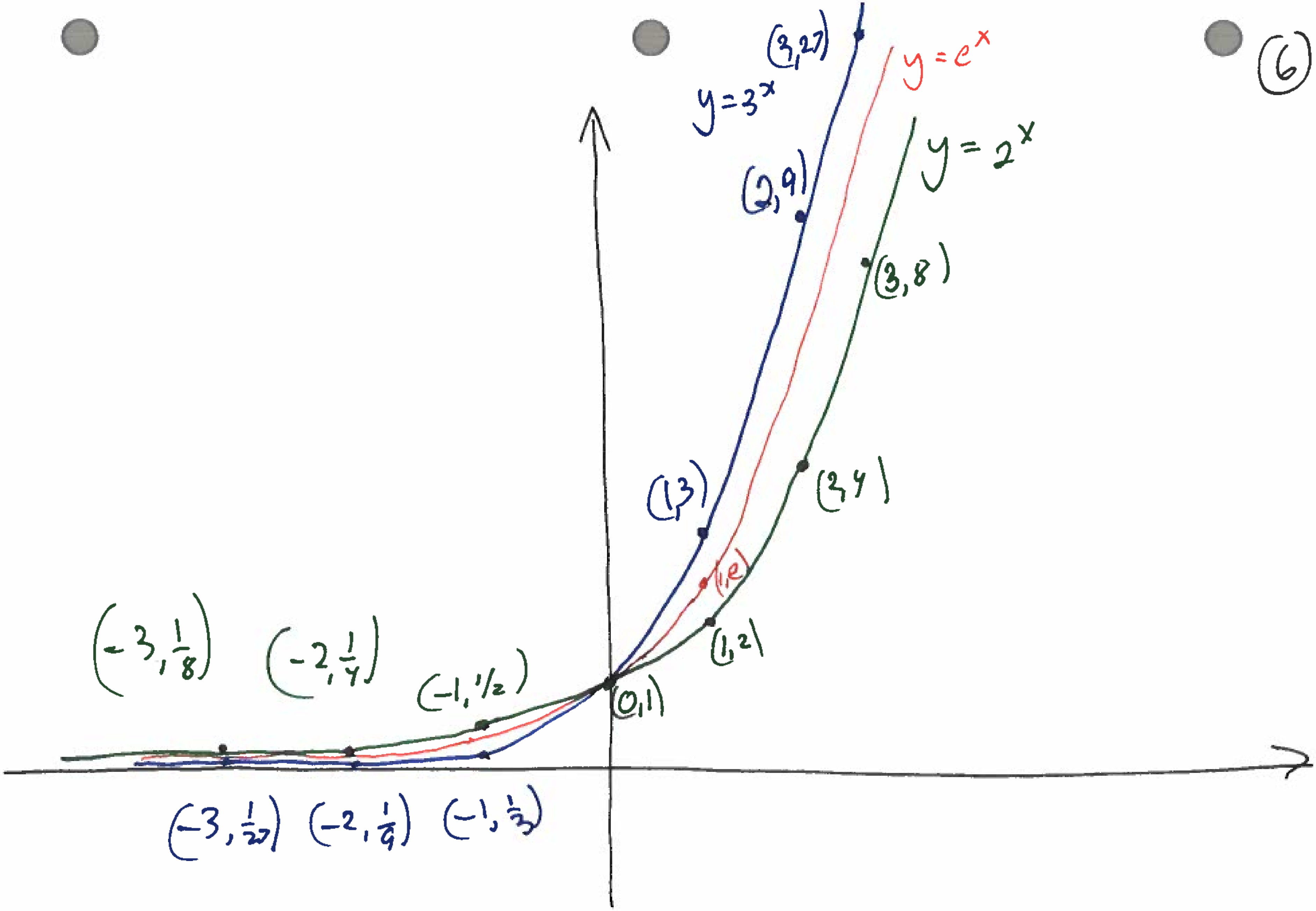
- The limit $\lim_{x \to 0} (1+x)^{1/x}$ does exist
- The value is denoted by the letter e.

$$e = \lim_{x \to 0} (1+x)^{1/x} \qquad \text{"Euler's number"}$$

- e is a real number.
- but e is irrational (e cannot be represented exactly by a fraction or a terminating decimal or even a repeating decimal)
- $e \approx 2.718$

Goal: Make graph of e using fact that $2<e<3$

x	2^x	e^x	3^x
$\vdots$	$\vdots$	$\vdots$	$\vdots$
-3	$2^{-3}=1/8$	$e^{-3}=\frac{1}{e^3}$	$3^{-3}=1/27$
-2	$2^{-2}=1/4$	$e^{-2}=\frac{1}{e^2}$	$3^{-2}=1/9$
-1	$2^{-1}=\frac{1}{2^1}=\frac{1}{2}$	$e^{-1}=\frac{1}{e}$	$3^{-1}=1/3$
0	$2^0=1$	$e^0=1$	$3^0=1$
1	$2^1=2$	$e^1=e$	$3^1=3$
2	$2^2=4$	e^2	$3^2=9$
3	$2^3=8$	e^3	$3^3=27$
$\vdots$	$\vdots$	$\vdots$	$\vdots$



$y = 3^x$
$(3, 27)$
$y = e^x$
$y = 2^x$
$(2, 9)$
$(3, 8)$
$(1, 3)$
$(2, 4)$
$(1, e)$
$(1, 2)$
$(0, 1)$
$(-3, \frac{1}{8})$
$(-2, \frac{1}{4})$
$(-1, \frac{1}{2})$
$(-3, \frac{1}{27})$
$(-2, \frac{1}{9})$
$(-1, \frac{1}{3})$

Properties of exponential functions $y = b^x$ with base $\cancel{b>0}$ $b>1$

- domain: all x
- range: all $y>0$
- contains famous points $(0,1)$ because $b^{(0)}=1$

  and $(1,b)$ because $b^{(1)}=b$
- graph is increasing
- $\lim_{x\to\infty} b^x = \infty$ right end goes up
- $\lim_{x\to -\infty} b^x = 0$ (horiz asymptote on left with equation $y=0$)