MATH 2301
Day 3

Fri Jan 20 lecture

Wednesday we discussed Section 1.3 Limits

Definition of limit

Informal Definition ← we will study and use this informal definition

Formal (Precise) Definition

We did examples of this type

(1) Given graph of f(x) → describe limit behavior

(2) given formula for f(x) → estimate the limit behavior

$$f(x) = \frac{x^2-2x-3}{x-3} = \frac{(x+1)(x-3)}{(x-3)}$$

we built tables of values

x	f(x)
2.9	computed
2.99	y values
2.999	

x getting closer & closer to 3 from left

y getting closer & closer to 4

$\lim_{x\to 3^-} f(x) = 4$

x	f(x)
3.1	computed y values
3.01	
3.01	

Similar observations

$\lim_{x\to 3^+} f(x) = 4$

Natural Question:

Is there a way to compute limits analytically, without making a table of values, and to get an exact answer, not a guess.

Good News

Yes! There is a way.

Bad News: The methods involve the formal, precise definition of Limit. Hard methods that we don't cover in MATH 2301.

(Take Advanced Calculus! It's really cool)

Good News

But in 2301, we'll use theorems about limits.

These are collected results (that are proven using the hard methods, but we won't bother with the proofs).

That brings us to

Section 1.4 Calculating Limits

We'll use those laws to find limits.

[Example #1] Using limit laws

Let $f(x) = \dfrac{x^2-2x-3}{x-3} = \dfrac{(x+1)(x-3)}{(x-3)}$

ⓐ compute $f(2)$

Solution $f(2) = \dfrac{(2+1)(2-3)}{(2-3)} = \dfrac{3\cancel{(-1)}}{\cancel{(-1)}} = 3$

ⓑ compute $\lim_{x\to 2} f(x)$

swap in formula for $f(x)$

Solution

$$\underbrace{\lim_{x\to 2} \underbrace{\frac{(x+1)(x-3)}{(x-3)}}_{\substack{\text{rational function}\\ \text{with domain}\\ \text{all } x \neq 3\\ \text{Notice } x=2\\ \text{is in the}\\ \text{domain}}}}_{\lim_{x\to 2} f(x)} \qquad \lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{(x+1)(x-3)}{(x-3)} = \underbrace{\frac{(2+1)(2-3)}{(2-3)}}_{f(2)} = 3$$

direct substitution property

(c) find $f(3)$

Solution ~~[illegible]~~

$$f(3) = \frac{(3+1)(3-3)}{(3-3)} = \frac{(4)(0)}{(0)} = \frac{0}{0}$$ does not exist DNE

no y value when x is 3.

(d) find $\lim_{x \to 3} f(x)$ Next page

swapped in the formula

(d) $\lim_{x\to 3} f(x) = \lim_{x\to 3} \frac{(x+1)\cancel{(x-3)}}{\cancel{(x-3)}} = \lim_{x\to 3} x+1 = 3+1 = 4$

this limit is an "indeterminate form" because if we substitute in $x=3$, we get $\frac{0}{0}$

polynomial function

no longer an indeterminate form

Direct substitution property

observe $x \to 3$
that tells us that $x \neq 3$
So $x-3 \neq 0$
So we canceled $\frac{x-3}{x-3}$

The most important concept of 1st month of Calculus

Example

Consider the expression

$$\frac{\frac{1}{5}+\frac{1}{x}}{5+x}$$

if we substitute in $x=-5$, we get

$$\frac{\frac{1}{5}+\frac{1}{(-5)}}{5+(-5)} = \frac{\frac{1}{5}-\frac{1}{5}}{5-5} = \frac{0}{0} \quad \underline{\text{undefined}}$$

But now, take the limit

rewrite in easier form

$$\lim_{x\to -5} \frac{\frac{1}{5}+\frac{1}{x}}{5+x} = \lim_{x\to -5} \frac{1}{5+x}\left[\frac{1}{5}+\frac{1}{x}\right]$$ still indeterminate

indeterminate form

$$= \lim_{x\to -5} \frac{1}{5+x}\left[\frac{1}{5}\left(\frac{x}{x}\right)+\frac{1}{x}\left(\frac{5}{5}\right)\right]$$ still indeterminate

get common denominator

$$= \lim_{x\to -5} \frac{1}{5+x}\left[\frac{x+5}{5x}\right]$$

$$= \lim_{x\to -5} \frac{\cancel{(5+x)}}{\cancel{(5+x)}\, 5x}$$ still indeterminate

most important concept of 1st month

$\boxed{\text{Since } x\to -5, \text{ we know } x\neq -5, \text{ so } 5+x\neq 0 \text{ so we can cancel } \frac{5+x}{5+x}}$

$$= \lim_{x\to -5} \frac{1}{5x}$$

rational function with −5 in its domain, so we can use the Direct Substitution Property

$$= \frac{1}{5(-5)} = -\frac{1}{25}$$

we found that

$$\lim_{x \to -5} \frac{\frac{1}{5} + \frac{1}{x}}{5 + x} = \cdots = -\frac{1}{25}$$