MATH 2301 Section 110 (Barsamian) Day 32 (Mon March 20, 2023)

Section 4.1 Max + Min Values

Definition Absolute Max + Min Values (they are y-values)

Words: Absolute Max value of function f on domain D.

meaning: A number $f(c)$ such that $c \in D$ and ~~such~~ that $f(c) \geq f(x)$ for all x in D.

a y value

epsilon

c is an element of the set D

$x \in D$.

Words: Absolute Min Value of f

meaning: A number $f(c)$ such that $c \in D$ and $f(c) \leq f(x)$ for all $x \in D$

Definition of Local Max and Local Min

Words: Local max value of a function f on a domain D.

Meaning: a number $f(c)$ such that $c \in (a,b) \subset D$ and $f(c) \geq f(x)$ for all ~~$x \in D$~~, $x \in (a,b)$

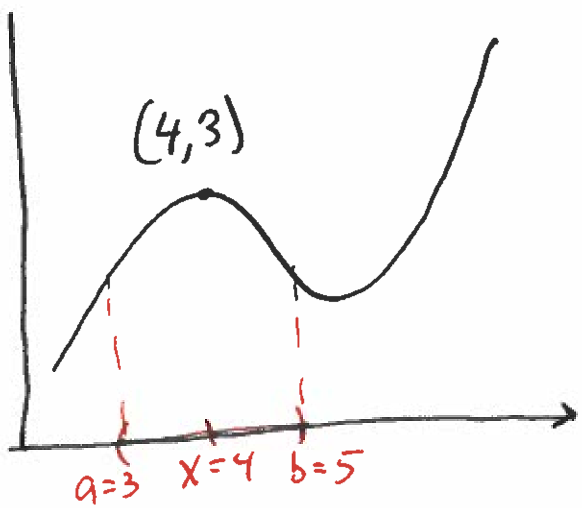
y value

is an element of

is a subset of

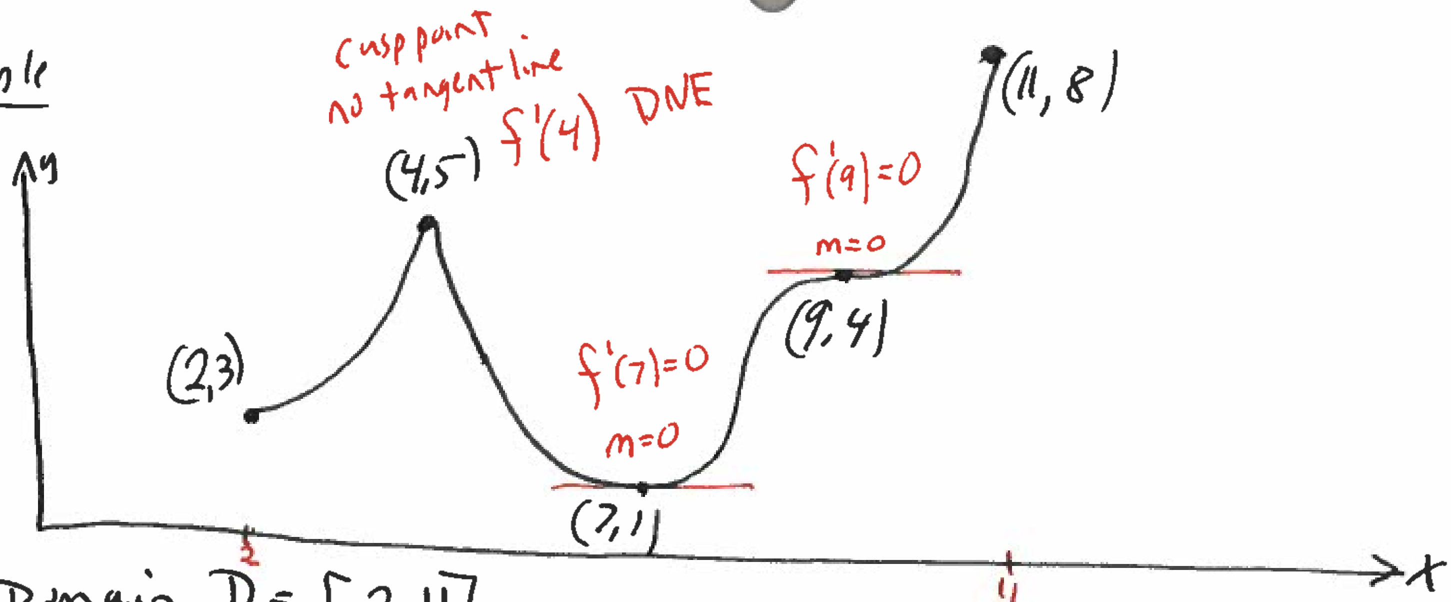
Words: Local min

meaning: - - - - - - - - - - $f(c) \leq f(x)$ for all $x \in (a,b)$



$f(4)=3$ is a local max value of $f(x)$

Example



On Domain $D = [2, 11]$

Absolute max value is $y=8$ (occurs at $x=11$). That is abs max value is $f(11) = 8$.

Absolut min value is $f(7) = 1$

Local max values

Local max value $f(4) = 5$

Notice: $f(11)=8$ is not qualified to be called a local max because it is an endpoint

Local min values: $f(7) = 1$

On Domain $(2, 11)$ (not including endpoints)

Absolute max value: there is no absolute max!

Absolute min value: $f(7) = 1$

Local max value: $f(4) = 5$

Local min value $f(7) = 1$

Observe: Sometimes, Absolute Max or Min Value is also a Local Max or Local min value, but not always

Extreme Value theorem

If function f is continuous on a closed interval $[a, b]$

then f will have ~~as~~ both an abs max value $f(c)$
and an abs min value $f(d)$ for some $c, d \in [a, b]$

Fermat's Theorem

If f has a local max or local min at $x = c$ then $f'(c) = 0$ or $f'(c)$ DNE

Definition of Critical Number

Words: Critical Number for a function f is an $x = c$ such that both of these requirements are met:

- $f(c)$ exists (that is $x = c$ is in the domain of f)
- $f'(c) = 0$ or $f'(c)$ DNE

Examples of finding critical Numbers

Let $f(x) = \dfrac{x}{x^2+1}$ Find the critical numbers

Observe: denominator will never be 0 because $x^2+1 \geq 1$

So domain is all x values. All real numbers.

Find $f'(x)$

$$f'(x) = \frac{d}{dx}\left(\frac{x}{x^2+1}\right) = \frac{\left(\frac{d}{dx}x\right)(x^2+1) - x\left(\frac{d}{dx}x^2+1\right)}{(x^2+1)^2} =$$

$$= \frac{(1)(x^2+1) - x(2x)}{(x^2+1)^2}$$

note: cannot cancel x^2+1 here!!

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{-x^2+1}{(x^2+1)^2}$$

$$= \frac{-(x^2-1)}{(x^2+1)^2} = -\frac{(x+1)(x-1)}{(x^2+1)^2}$$

We have found $f'(x) = -\dfrac{(x+1)(x-1)}{(x^2+1)^2}$

Look for critical numbers:

- x values that cause $f'(x)$ to <u>not</u> <u>exist</u>?

  there are none, because $x^2+1 \geq 1$ so denominator ≥ 1

- x values that cause $f'(x) = 0$?

  Set $f'(x) = 0$ and solve for x

$$-\frac{(x+1)(x-1)}{(x^2+1)^2} = 0$$

remember: a fraction $\frac{a}{b} = 0$ only when $a = 0$ and $b \neq 0$

we need to find x values such that

$$(x+1)(x-1) = 0$$

the solutions are $x = -1$ and $x = 1$

Conclusion for $f(x) = \dfrac{x}{x^2+1}$

the critical numbers are

$x = -1$ and $x = 1$

because they both are in domain and

cause $f'(x) = 0$