MATH 2301 Section 110 (Barsamian) Day #34 (Wed March 22) ①

- Today: Section 4.1 Max + Min Values
- Friday: Section 4.2 The Mean Value Theorem
  Quiz Q6 covering Section 4.1

Section 4.1 Max & Min Values

Remember from Monday

Definition A critical number for a function $f(x)$ is an $x=c$ that satisfies these two requirements

- $f(c)$ exists ($x=c$ is in the domain of $f(x)$)
- $f'(c)=0$ or $f'(c)$ DNE

horizontal tangent line

either no tangent line because of sharp cusp in graph or vertical tangent line.

Why is the concept of a Critical Number useful? (2)

Local Max + Local Min can only occur at x values that are critical numbers for the function f and that are not endpoints of the domain.

So, to find local max + min, start by looking for critical numbers.

We will return to this idea later.

<u>Absolute Max + Min</u> can only occur at these kinds of important x values

- x values that are endpoints of the domain
- x values that are <u>critical</u> <u>numbers</u> in the domain.

So if you want to find absolute max + min, finding critical numbers is part of that search.

Recall Extreme Value Theorem (from Monday)

If a function $f(x)$ is continuous on a closed interval $[a, b]$,

~~we~~ there will be both an abs max and an abs min on the interval.

The Closed Interval Method

used for finding the absolute max value and absolute min value for a function $f(x)$ that is continuous on a closed interval $[a,b]$

Step 1 Confirm that there is a closed interval and that $f(x)$ is continuous on that interval

Step 2 Find the critical numbers for $f(x)$

Step 3 Make a list of important x values in increasing order.

important x values	Corresponding y values
$x=a$ (endpoint)	$f(a)$
$x=c_1$ } critical numbers in the interval	$f(c_1)$
$x=c_2$ } critical numbers in the interval	$f(c_2)$
$\vdots$	$\vdots$
$x=b$ (endpoint)	$f(b)$

Step 4 Compute the corresponding y values

Step 5 Identify the max & min y values on this list. Write a clear conclusion in a sentence.

<u>Example</u> Find absolute max + min values of $f(x) = x^4 - 6x^2 + 5$ on the interval $[-3,3]$

<u>Solution</u> Use Closed Interval Method

<u>Step 1</u> The interval <u>is</u> closed and f is continuous because it is a polynomial

<u>Step 2</u> Find critical numbers

$$f'(x) = \frac{d}{dx}\left(x^4 - 6x^2 + 5\right) = 4x^3 - 12x$$

Notice: $f'(x)$ is also a polynomial, so there are <u>no</u> x values that cause $f'(x)$ to not exist.

Set $f'(x) = 0$ and solve for x

$$0 = f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$$

$$= 4x(x+\sqrt{3})(x-\sqrt{3})$$

$$= 4(x+\sqrt{3})\cdot x \cdot (x-\sqrt{3})$$

Solutions: $x = -\sqrt{3}$ $x = 0$ $x = \sqrt{3}$ critical numbers

Step 3

Step 4

important x values		Corresponding y values $f(x) = x^4 - 6x^2 + 5$
$x = -3$	endpoint	$f(-3) = (-3)^4 - 6(-3)^2 + 5 = 81 - 54 + 5 = 32$
$x = -\sqrt{3}$	crit	$f(-\sqrt{3}) = (-\sqrt{3})^4 - 6(-\sqrt{3})^2 + 5 = 9 - 6 \cdot 3 + 5 = -4$
$x = 0$	crit	$f(0) = (0)^4 - 6(0)^2 + 5 = 5$
$x = \sqrt{3}$	crit	$f(\sqrt{3}) = (\sqrt{3})^4 - 6(\sqrt{3})^2 + 5 = 9 - 6 \cdot 3 + 5 = -4$
$x = 3$	endpoint	$f(3) = (3)^4 - 6(3)^2 + 5 = 81 - 54 + 5 = 32$

Abs max value is $y = 32$. (It occurs at $x = -3$ and $x = 3$)

Abs min value is $y = -4$. (It occurs at $x = -\sqrt{3}$ and $x = \sqrt{3}$)

End of Example 1

Frick, Frack, and Wacky Jack Solved this problem differently

Frick's solution Just check *all* the x values!

x	y
-3	$y = \text{----} = 32$
-2	$y = \text{----} = -3$
-1	$y = \text{----} = 0$
0	$y = \text{----} = 5$
1	$y = \text{----} = 0$
2	$y = \text{----} = -3$
3	$y = \text{----} = 32$

Frick says
abs max value is $y = 32$
abs min value is $y = -3$

invalid method

Frick happened to get correct y value for max
but they got wrong y value for min

Invalid method: Does ~~dot~~ not conside endpoints

Frack's Solution

$$f(x) = x^4 - 6x^2 + 5$$

$$f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$$

Critical numbers $x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$

important x values	
$x = -\sqrt{3}$	$y = f(-\sqrt{3}) = \ldots\ldots = [illegible]$ -4
$x = 0$	$y = f(0) = \ldots\ldots = 5$
$x = 3$	$y = f(\sqrt{3}) = \ldots\ldots = [illegible]$ -4

correct answer

incorrect answer

Abs max $y = 5$, Abs min $y = -4$

invalid method Does not consider endpoints.

Wacky Jack's Solution

$f(x) = x^4 - 6x^2 + 5$

$f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$

critical numbers $x = -\sqrt{3},\ 0,\ \sqrt{3}$

Wacky jack computed f' values, not f values

important x values	
-3 end	$\cancel{4(-3)^3 - 12(-3) = -108 + 36 = -72}$
$-\sqrt{3}$ crit	$\cancel{4(-\sqrt{3})((-\sqrt{3})^2 - 3) = 4(-\sqrt{3})(0) = 0}$
0 crit	$\cancel{4(0)(0^2 - 3) = 0}$
$\sqrt{3}$ crit	$\cancel{4(\sqrt{3})((\sqrt{3})^2 - 3) = 4\sqrt{3}(0) = 0}$
3 end	$\cancel{4(3)(3^2 - 3) = 12(6) = 72}$

max $y = 72$ min $y = -72$.

invalid. Did not use f(x) to find y values.