MATH 2301 Section 110 (Barsamian) Day 36 (Mon March 27)

Comments on X2 (See printed Solutions)

[1] rewrite!

[2] chain rule + product rule necessary

[3] product rule + chain rule necessary

[4] cancel right away!

Identify known quantities + rates, Identify which unknown we need to find

[5] Chain Rule

[6] See my Solutions

[7] (a) find t, then find height at that time

(b) find t, then find velocity at that time

[8] $\ln(1) = 0$!!

Chain rule

Section 4.3 Derivatives and the Shapes of Graphs

The Increasing/Decreasing Test

(a) If $f'(x) > 0$ on an interval, then f is increasing on that interval.

(b) If $f' < 0$ on an interval, then f decreasing on interval

The First Derivative Test

Suppose $x = c$ is a critical number of a continuous function f.

(a) If f' changes from pos to neg at $x = c$ then f has local max ~~f(c)~~. value $f(c)$

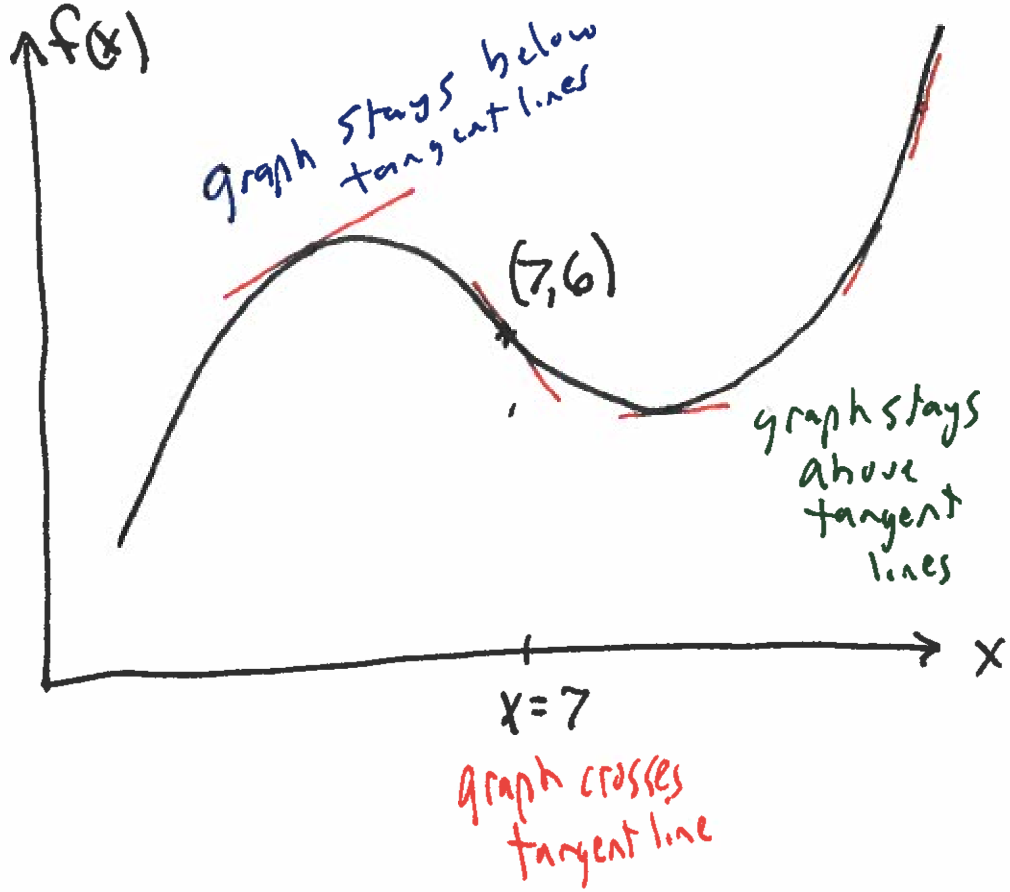
(b) If f' changes from neg to pos at $x = c$ then f has local min value $f(c)$

(c) If f' does not change sign at $x = c$ then f does not have local extremum at $x = c$

Definition of concavity

If a graph lies above its tangent lines on an interval I, then we say the function concave up on that interval

If a graph lies below its tangent lines on an interval, then we say the function is concave down on that interval



f is concave up on the interval $(7, \infty)$

f concave down on $(-\infty, 7)$

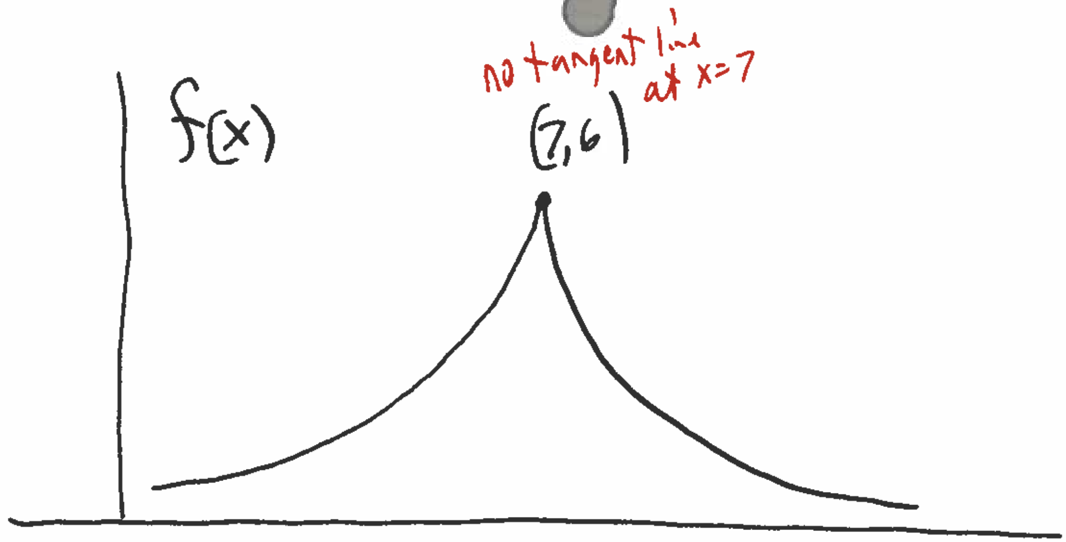
Definition of inflection point

An *inflection point* for a function f is

- a point on the graph of f
- where the concavity changes from up to down or down to up

Example on previous graph, the point $(7, 6)$ is an inflection point.

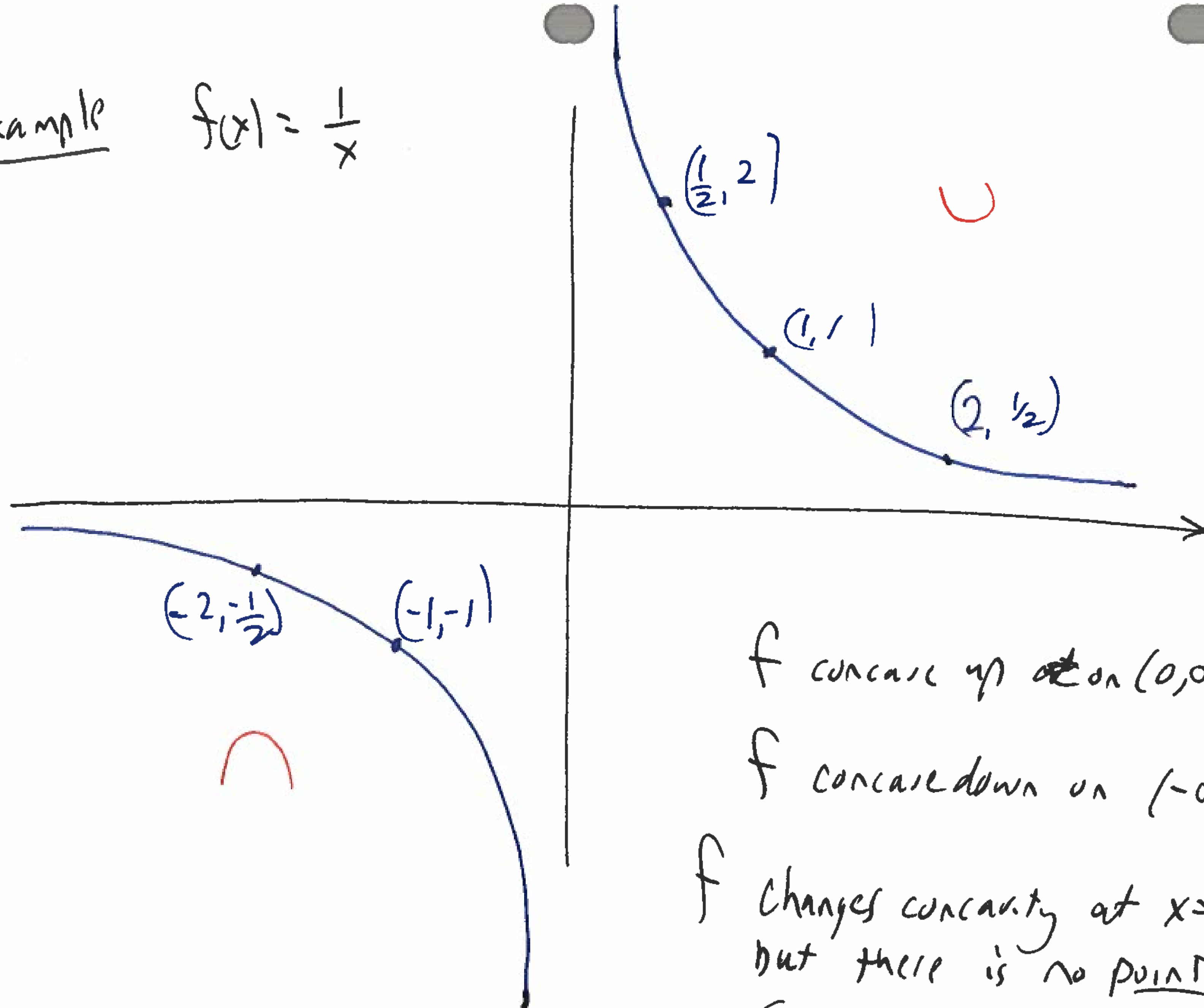
Example



f concave up on $(-\infty, 7)$ and $(7, \infty)$

We <u>cannot</u> say f concave up on (∞,∞) because f doesn't have tangent line there.

Example $f(x) = \frac{1}{x}$



f concave up ~~at~~ on $(0, \infty)$

f concave down on $(-\infty, 0)$

f changes concavity at $x = 0$, but there is no point there, so no inflection point.

Concavity Test

If $f''(x) > 0$ for all x in an interval, then f is concave up on that interval

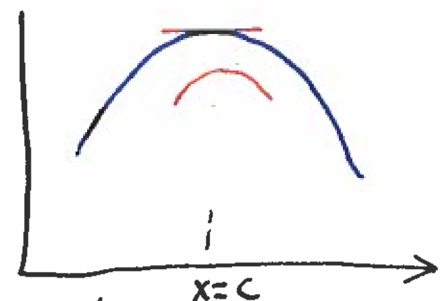
If $f''(x) < 0$ for all x in interval then f concave <u>down</u> on that interval

Second Deriv Test for Local Extrema

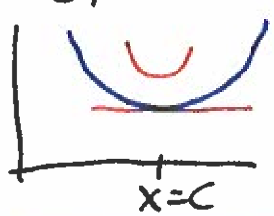
Suppose $f''(x)$ is continuous near $x = c$

If $f'(c) = 0$ and $f''(c) < 0$ then f has local max value $f(c)$

horiz tangent — concave down



If $f'(c) = 0$ an $f''(c) > 0$ then f has local <u>min</u> at $f(c)$



Examples using these facts

[Example 1] Let $f(x) = -x^4 + 4x^3 + 7$

(a) Find intervals where f increasing or decreasing

Solution Strategy: Find $f'(x)$

Study sign of $f'(x)$

make conclusions about $f(x)$.

$$f'(x) = -4x^3 + 12x^2 = -4x^2(x-3)$$

↑ factored

$f'(x) = 0$ when $x=0$ or $x=3$

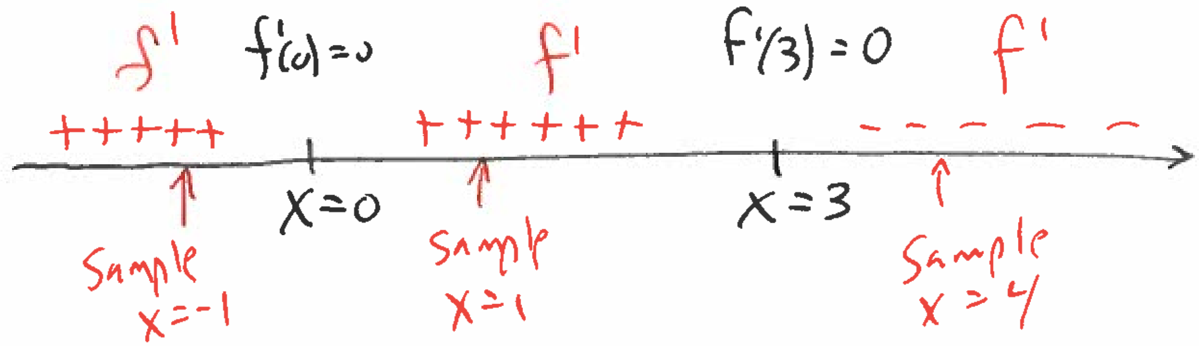
(these are the critical numbers of $f(x)$)

$f'(x)$ is a polynomial, so f' is continuous

Sign can only change at x values where $f'(x)=0$

Make Sign chart for $f'(x)$

(1) Put critical numbers on a numberline, indicate behavior of $f'(x)$ at these numbers.



(2) choose sample numbers in intervals and compute sign of $f'(x)$

$f'(-1) = \quad -4(-1)^2((-1)-3) = -4(1)(-4) = $ pos

use factored version! Easier

$f'(1) = -4(1)^2((1)-3) = -4(1)(-2) = $ pos

$f'(4) = -4(4)^2((4)-3) = -4(16)(1) = $ neg

<u>conclude</u>

f increasing on $(-\infty, 0)$ because f' pos

f increasing on $(0, 3)$ because f' pos

f increasing on interval $(-\infty, 3)$

f decreasing on $(3, \infty)$