Math 2301 Section 110 (Barsamian) Meeting #38 (Wed Mar 29)

- Pick up Graded Work
- Pick up Handout on Graphing Strategy
- Today: Section 4.4 Curve Sketching
- Friday: Section 4.5 Optimization
  Quiz Q7 Covering Section 4.2, 4.3

Today Section 4.4 Curve Sketching

Refer to handout on Curve Sketching "Graphing Strategy"

Bursamian

Reference R05: Graphing Strategy

General Idea:

- Start with simplest, least sophisticated analysis, then proceed to more sophisticated.
- If a step is not easy (for instance, if it is not clear how to factor $f(x)$), then skip that step.

Step 1. Analyze $f(x)$.

- Find the y-intercept
- Find the x-intercepts.
- Determine the end-behavior (Are there horizontal asymptotes? Or do the ends of the graph go up or down?) by finding $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Determine location of any vertical asymptotes
- Make a sign chart for f and use it to determine where f is positive, negative, or zero.

Step 2. Analyze $f'(x)$.

- Find $f'(x)$, factor it, and then find the partition numbers for $f'(x)$.
- Construct a sign chart for $f'(x)$ and use it to determine the x coordinates where graph of f has a horizontal tangent line, the intervals on which f is increasing and decreasing, and the x coordinates of all relative maxima and minima.
- Find the y coordinates of all relative maxima and minima.

Step 3. Analyze $f''(x)$.

- Find $f''(x)$, factor it, and then find the partition numbers for $f''(x)$.
- Construct a sign chart for $f''(x)$ and use it to determine the intervals on which f is concave up and concave down, and to find the x coordinates of all inflection points.
- Find the y coordinates of all inflection points.

Step 4: Sketch the graph of f.

- Draw any asymptotes as dotted lines, and label them with their line equations.
- Plot the axis intercepts, relative maxima and minima, and inflection points, and label them with their (x, y) coordinates.
- Using the other information from steps 1, 2, and 3, draw the graph.

Example #1 Use graphing strategy to sketch a graph of

$$f(x) = -x^4 + 4x^3$$

Solution

Step 1 Analyze $f(x)$

y intercept: (set $x=0$, find y)

$$y = f(0) = -(0)^4 + 4(0)^3 = 0$$

y intercept at $(x,y) = (0,0)$

x intercepts: (set $y=0$, solve for x)

$$0 = y = f(x) = -x^4 + 4x^3$$

$$0 = x^3(-x+4) \quad \text{factored}$$

Solutions: $x=0$, $x=4$ X intercepts $(x,y)=(0,0)$ and $(x,y)=(4,0)$

End Behavior: even degree polynomial
negative leading coefficient
So both ends of graph go down. ↙ ↘

<u>Vertical Asymptotes</u>? None because domain of f is all real numbers

f <u>polynomial</u>, so always continuous

continuous

both ends go down

<u>Sign Chart for</u> $f(x) = x^3(-x+4) = -x^3(x-4)$

(f is continuous, so it can only change sign at its x intercepts.)

$f(0)=0$ $\qquad$ $f(4)=0$

$- - - - - - \quad + + + + \quad - - - -$

$x=0$ $\qquad$ $x=4$

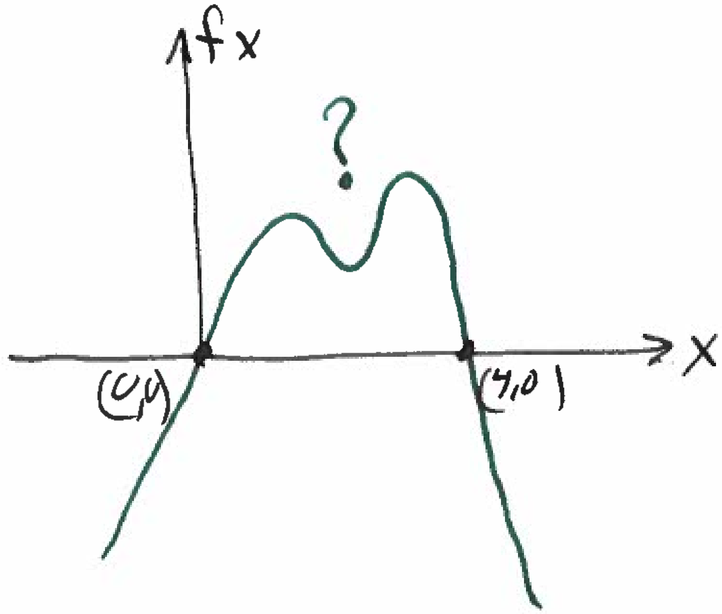
Sample $x=-1$ $\qquad$ Sample $x=1$ $\qquad$ Sample $x=5$

$f(-1) = -(-1)^3((-1)-4) = -(-1)(-5) = $ neg

$f(1) = -(1)^3((1)-4) = -(1)(-3) = $ pos

$f(5) = -(5)^3(5-4) = -(125)(1) = $ neg

Crude graph after Step 1



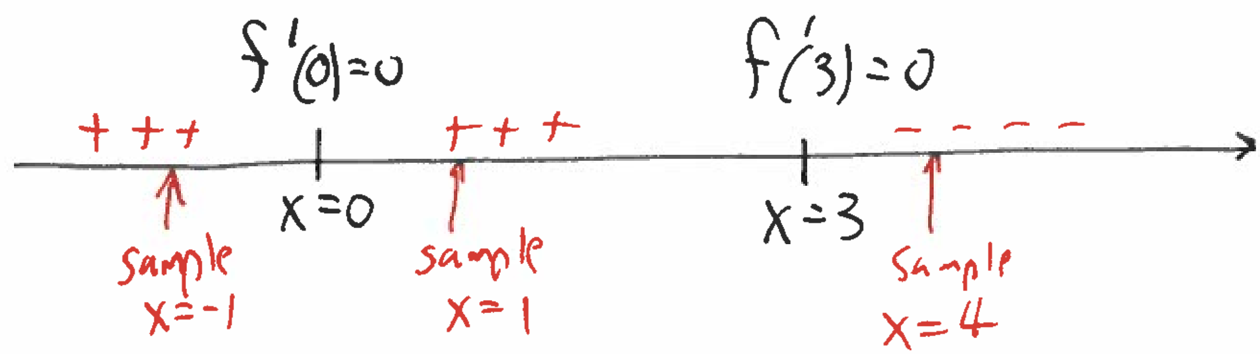
Step 2 Analyze $f'(x) = -4x^3 + 12x^2 = 4x^2(-x+3)$ (6)

$$= -4x^2(x-3)$$

Partition numbers for $f'(x)$? x-values that cause $f'(x) = 0$ or $f'(x)$ not exist

$$x = 0,\ x = 3$$

Sign chart for $f'(x) = -4x^2(x-3)$



From monday $f'(-1) = -4(-1)^2(-1-3) = \text{pos}$

$f'(1) = -4(1)^2(1-3) = \text{pos}$

$f'(4) = -4(4)^2(4-3) = \text{neg}$

Conclusions

f has horiz tangent at $x=0$, $x=3$ because $f'=0$ there

f increasing on $(-\infty, 3)$

f decreasing on $(3, \infty)$

f has local max at $x=3$ because f' changes pos $\to$ zero $\to$ neg

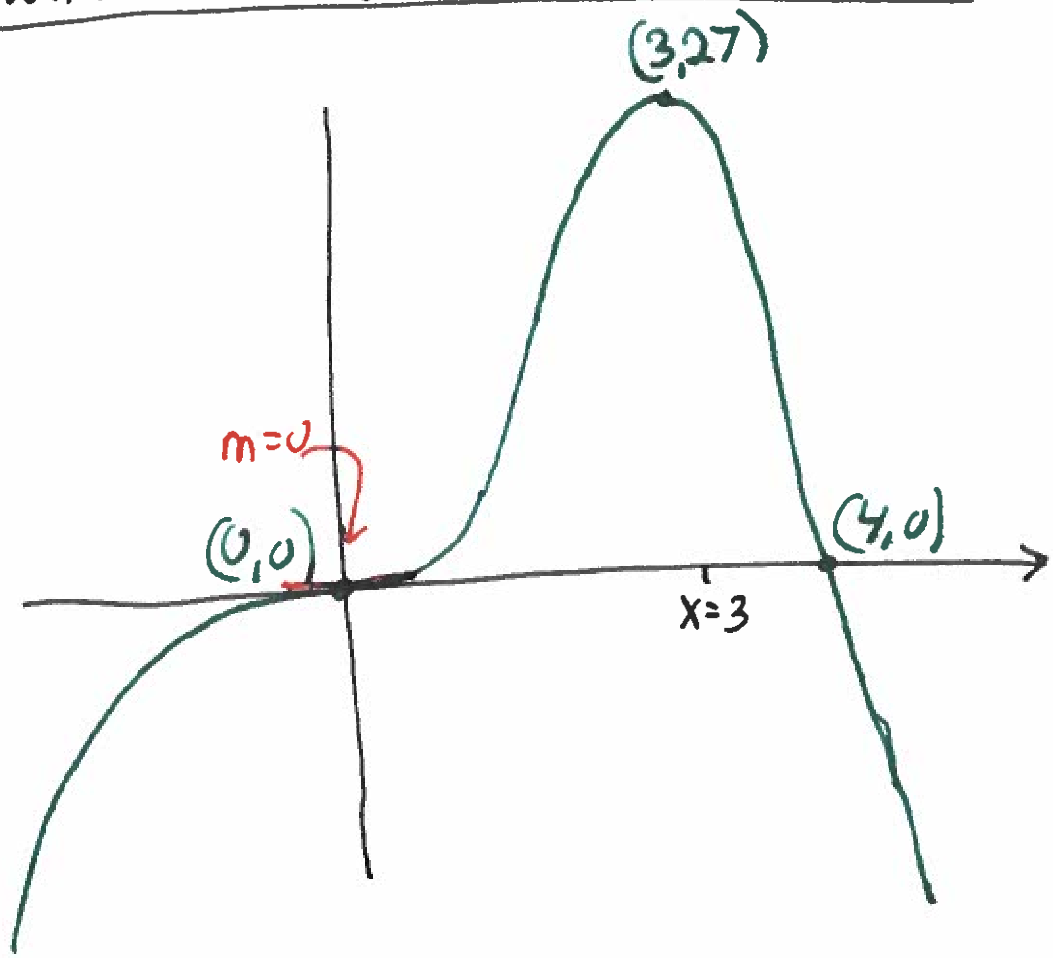
No local mins

Corresponding y coordinate

$$f(3) = -(3)^4 + 4(3)^3 = -81 + 4(27) = -81 + 108 = 27$$

Local max value is $f(3) = 27$

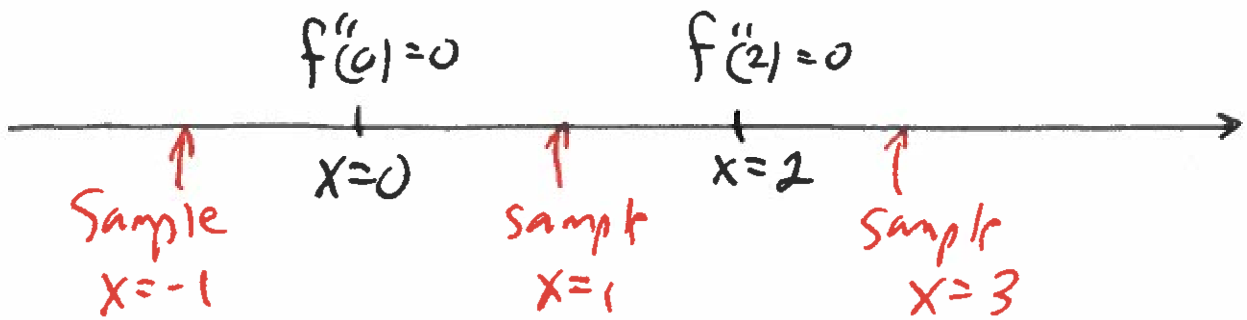
Pause + make graph after step 2



Step 3 Analyze $f''(x) = -12x^2 + 24x = 12x(-x+2)$
$$= -12x(x-2)$$

partition numbers $x=0$, $x=2$ cause $f''(x) = 0$

Sign chart for $f''(x) = -12x(x-2)$



$f''(-1) = -12(-1)(-1-2) = (-)(-)(-) = \text{neg}$

$f''(1) = -12(1)(1-2) = (-)(+)(-) = \text{pos}$

$f''(3) = -12(3)(3-2) = (-)(+)(+) = \text{neg}$

f concave up on $(0,2)$ because f'' pos

f concave down on $(-\infty,0)$ and $(2,\infty)$ because f'' neg

Inflection points at $x=0$, $x=2$ because concavity changes and there are points on graph of $f(x)$

Corresponding y values $f(x) = -x^4 + 4x^3$

$f(0) = 0$ inflection point $(x, y) = (0, 0)$

$f(2) = -(2)^4 + 4(2)^3 = -16 + 4(8) = -16 + 32 = 16$

inflection point $(x, y) = (2, 16)$

Step 4 Graph