MATH 2301 Section 110 (Barsamian) Meeting #39 (Fri March 31, 2023)

- Pick up Graded Work
- If you don't already have one, pick up a copy of the Graphing Strategy
- Today
  - ~~At~~ One More Example from 4.4 Curve Sketching
  - Section 4.5 Optimization
  - Quiz Q7
- Next Friday: Exam X3 Covering Chapter 4

Reference R05: Graphing Strategy

General Idea:

- Start with simplest, least sophisticated analysis, then proceed to more sophisticated.
- If a step is not easy (for instance, if it is not clear how to factor $f(x)$), then skip that step.

Step 1. Analyze $f(x)$.

- Find the y-intercept
- Find the x-intercepts.
- Determine the end-behavior (Are there horizontal asymptotes? Or do the ends of the graph go up or down?) by finding $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$.
- Determine location of any vertical asymptotes
- Make a sign chart for f and use it to determine where f is positive, negative, or zero.

Step 2. Analyze $f'(x)$.

- Find $f'(x)$, factor it, and then find the partition numbers for $f'(x)$.
- Construct a sign chart for $f'(x)$ and use it to determine the x coordinates where graph of f has a horizontal tangent line, the intervals on which f is increasing and decreasing, and the x coordinates of all relative maxima and minima.
- Find the y coordinates of all relative maxima and minima.

Step 3. Analyze $f''(x)$.

- Find $f''(x)$, factor it, and then find the partition numbers for $f''(x)$.
- Construct a sign chart for $f''(x)$ and use it to determine the intervals on which f is concave up and concave down, and to find the x coordinates of all inflection points.
- Find the y coordinates of all inflection points.

Step 4: Sketch the graph of f.

- Draw any asymptotes as dotted lines, and label them with their line equations.
- Plot the axis intercepts, relative maxima and minima, and inflection points, and label them with their (x, y) coordinates.
- Using the other information from steps 1, 2, and 3, draw the graph.

One more Example from Section 4.4 Curve Sketching

Let $f(x) = xe^x$ (function from Quiz Q6 [2])

Use Graphing Strategy to sketch graph of $f(x)$

Step 1 Analyze $f(x)$

y intercept: set $x=0$, find y

$$y = f(0) = 0 \cdot e^{(0)} = 0 \cdot 1 = 0$$

y intercept at $(x,y) = (0,0)$

X intercepts: set $y=0$, find x

$$0 = x e^{(x)}$$

this term is zero only when $x=0$

$e^{(x)}$ is never 0!!

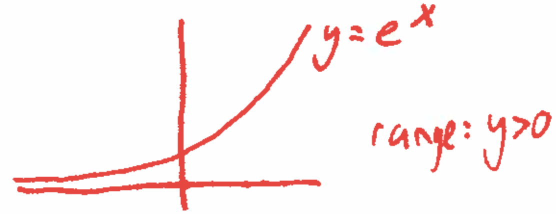
So the only solution is $x=0$

So the only x intercept is $(x,y) = (0,0)$

Zero Product Property

If $a \cdot b = 0$

then $a=0$ or $b=0$ (or both)



<u>End behavior</u> (what happens on graph as $x \to \infty$ (right end) and $x \to -\infty$ (left end)?)

Compute limits

~~right end~~ $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x e^{(x)} = \infty$

right end

x getting huge, positive without bound

huge, positive

also getting huge, positive

product is getting huge and positive

right end of graph goes up

~~left end~~ $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x e^{(x)}$ = not sure

left end

x getting huge, negative without bound

huge, negative

positive but gets closer and closer to zero

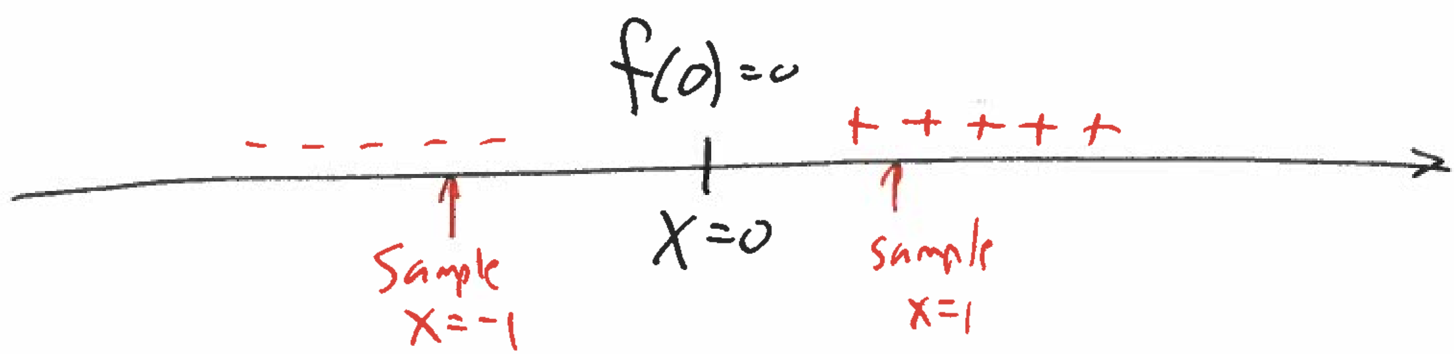
product will be negative

Not sure what the left end of graph is doing

Any vertical asymptotes for function $f(x)=xe^x$?

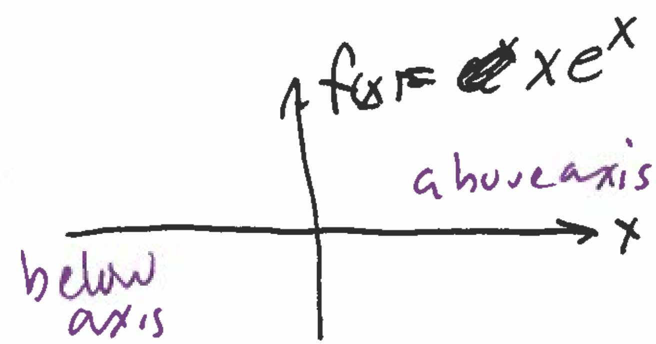
No, because domain is all x.

Sign chart for $f(x)=Xe^{(x)}$



$$f(-1)=\underbrace{(-1)}_{neg}\cdot\underbrace{e^{(-1)}}_{pos}=-1\cdot\frac{1}{e^1}=-\frac{1}{e}=\text{negative}$$

$$f(1)=\underbrace{(1)}_{pos}\cdot\underbrace{e^{(1)}}_{pos}=\text{pos}$$



Step 2 Analyze $f'(x)$

power rule

$$f'(x) = \frac{d}{dx}\left(xe^{(x)}\right) \overset{\uparrow}{=} \left(\frac{dx}{dx}\right)e^{(x)} + x\cdot\left(\frac{d}{dx}e^{x}\right) = (1)\cdot e^{(x)} + x\cdot(e^{x})$$

product rule

exponential function rule

$$\frac{d}{dx}e^{x} = e^{x}$$

$$= (1)e^{(x)} + xe^{(x)}$$

$$\overset{\text{factor}}{=} (1+x)e^{(x)}$$

Find partition numbers for $f'(x)$

Domain of $f'(x)$ is all real numbers

So there are <u>no</u> x values that cause $f'(x)$ to not exist.

Look for numbers ~~x~~ x that cause $f'(x) = 0$

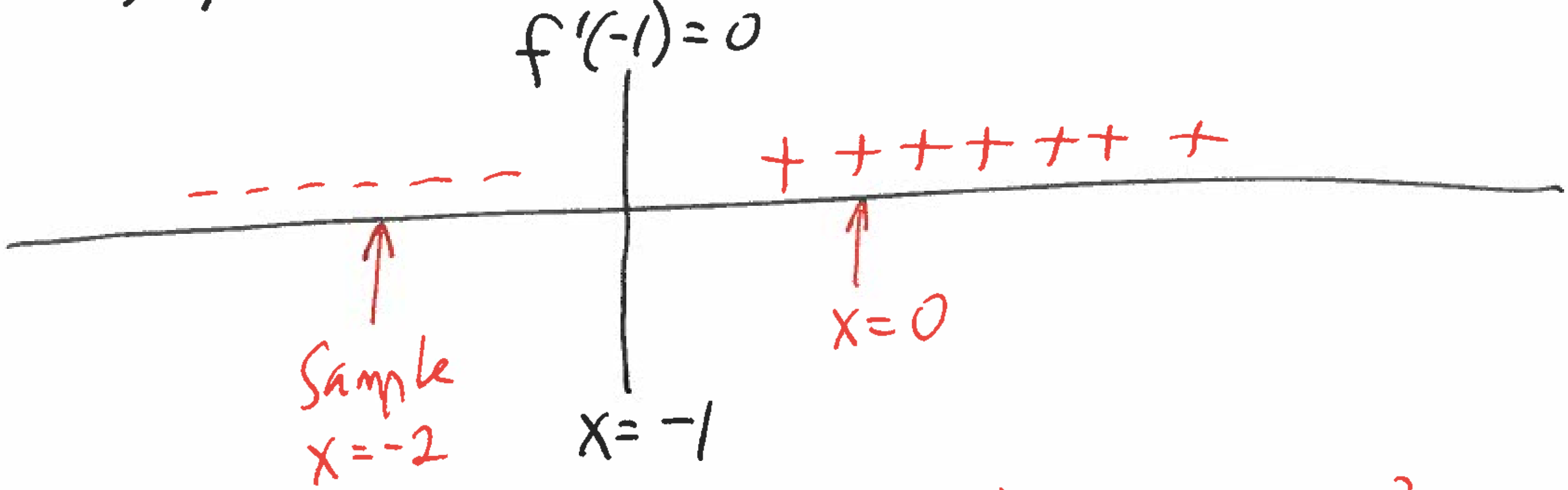
$$0 = \underbrace{(1+x)}_{\substack{\uparrow \\ \text{only zero when} \\ x=-1}} \overset{\text{never zero}}{e^{(x)}}$$

Only partition numbers for $f'(x) = (1+x)e^{(x)}$ are $x=-1$ because $f'(-1) = 0$

This tells us that the only critical number for $f(x) = xe^{(x)}$ is $x=-1$

(Quiz 6 problem [2])

Sign chart for $f'(x) = (1+x)e^{x}$



$$f'(-2) = (1+(-2))e^{(-2)} = \underbrace{(-1)}_{neg}\cdot \underbrace{e^{-2}}_{pos} = neg$$

$$f'(0) = (1+(0))e^{(0)} = 1\cdot 1 = pos$$

So $f(x)$ is decreasing on $(-\infty, -1)$ because f' neg

$f(x)$ is increasing on $(-1, \infty)$ because f' pos

local min (absolute min) at $x=-1$

y coordinate is $f(-1) = (-1)e^{(-1)} = -1\cdot\frac{1}{e} = -\frac{1}{e}$

Absolute min at $(x,y) = (-1, 1)$

Step 3 Analyze $f''(x)$

$$f''(x) = \frac{d}{dx} f'(x) = \frac{d}{dx}\left((1+x)e^x\right)$$

$$= \left(\frac{d}{dx} 1+x\right) \cdot e^x + (1+x)\frac{d}{dx} e^x$$

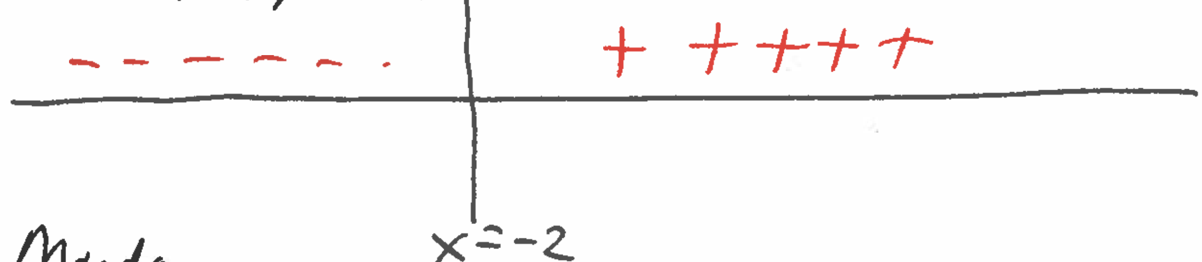
$$= 1\,e^x + (1+x)e^{(x)}$$

factor
$$= (1+1+x)e^x$$

$$= (2+x)e^x$$

$f''(x) = 0$ only when $x = -2$

Signchart for $f''(x)$ $\quad f''(-2) = 0$



Finish on Monday