Day 4 (Mon Jan 23) Lecture on 1.4 Calculating Limits ①

Recall examples from friday

(Similar to 1.4 #17) $\underbrace{\lim_{x\to 3} \frac{(x+1)\cancel{(x-3)}}{\cancel{(x-3)}}}_{\text{indeterminate form}} = \underbrace{\lim_{x\to 3} x+1}_{\text{no longer indeterminate}} = 4$

since $x \to 3$ we know $x \neq 3$ so $x-3 \neq 0$

(Similar to 1.4 #25)

$\underbrace{\lim_{x\to -5} \frac{\frac{1}{5}+\frac{1}{x}}{5+x}}_{\text{indeterminate form}} = \text{ord} = \text{rewriting} \underbrace{\lim_{x\to -5} \frac{5+x}{5+x} \cdot \text{stuff}}_{\text{still indeterminate}} = \underbrace{\lim_{x\to -5} \text{stuff}}_{\text{no longer indeterminate}} = \frac{-1}{25}$

since $x \to -5$ we know $x \neq -5$ so $x+5 \neq 0$

Meeting Part 1 More Limits involving Indeterminate Forms

Similar example (Sim to 1.4 #17)

$$\lim_{h\to 0} \frac{(2+h)^3-8}{h} = \lim_{h\to 0} \frac{(2^3+3\cdot 2^2\cdot h+3\cdot 2\cdot h^2+h^3)-8}{h}$$ Still indet

$\frac{0}{0}$ indeterminate form

$$= \lim_{h\to 0} \frac{\cancel{8}+12h+6h^2+h^3\cancel{-8}}{h}$$ Still indet

$$= \lim_{h\to 0} \frac{12h+6h^2+h^3}{h}$$ Still indet

$$= \lim_{h\to 0} \frac{\cancel{h}(12+6h+h^2)}{\cancel{h}}$$ Still indet

Since $h\to 0$, we know $h\neq 0$, so we can cancel $\frac{h}{h}$

$$= \lim_{h\to 0} 12+6h+h^2$$ no longer indeterminate

Direct substitution property

$$= 12+6(0)+(0)^2$$

$$= 12$$

Example (Similar to L4 #21)

"rationalizing"

this is just the number 1

$$\underbrace{\lim_{h\to 0} \frac{\sqrt{49+h}-7}{h}}_{\frac{0}{0} \text{ indeterminate form}} \underset{\text{Trick I}}{=} \lim_{h\to 0} \frac{\sqrt{49+h}-7}{h}\left(\frac{\sqrt{49+h}+7}{\sqrt{49+h}+7}\right)$$

$$= \lim_{h\to 0} \frac{\sqrt{49+h}\cdot\sqrt{49+h} \cancel{-7\sqrt{49+h}} \cancel{+7\sqrt{49+h}} -7\cdot 7}{h(\sqrt{49+h}+7)}$$

$$= \lim_{h\to 0} \frac{\cancel{49}+h \cancel{-49}}{h(\sqrt{49+h}+7}$$

$$= \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{49+h}+7)}$$

still indeterminate

Since $h\to 0$, we know $h\neq 0$ so we can cancel $\frac{h}{h}$

$$= \lim_{h\to 0} \frac{1}{\sqrt{49+h}+7}$$

no longer indeterminate

Direct substitution

$$= \frac{1}{\sqrt{49+(0)}+7}$$

$$= \frac{1}{7+7}$$

$$= \frac{1}{14}$$

Meeting Part 2

Piecewise-Defined functions, Absolute Value, One-sided limits

Definition of absolute value Function

$$\boxed{|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}}$$

Piecewise-defined function

more ink

$$\begin{cases} |x| \text{ means } x & \text{when } x \geq 0 \\ |x| \text{ means } -x & \text{when } x < 0 \end{cases}$$

Turned around

$$\boxed{\begin{cases} \text{when } x \geq 0, \text{ symbol } |x| \text{ means } x \\ \text{when } x < 0, \text{ the symbol } |x| \text{ means } -x \end{cases}}$$

Example (similar to 1.4 #38)

$$\lim_{x \to -11} \frac{5x+55}{|x+11|}$$

($\frac{0}{0}$ indeterminate form)

Rewrite without absolute value

when $x+11 \geq 0$ symbol $|x+11|$ just means $x+11$

That is,

when $x \geq -11$ the symbol $|x+11|$ means $x+11$

when $x+11 < 0$ the symbol $|x+11|$ means $-(x+11)$

That is,

when $x < -11$, the symbol $|x+11|$ means $-(x+11)$

limit from the left

$$\lim_{x \to -11^-} \frac{5x+55}{|x+11|} = \lim_{x \to -11^-} \frac{5x+55}{-(x+11)}$$

this tells us $x < -11$

Swap in the appropriate formula

$$= \lim_{x \to -11^-} \frac{5\cancel{(x+11)}}{-\cancel{(x+11)}}$$

indeterminate

Since $x < -11$, we know $x \neq -11$, so $x+11 \neq 0$, so we can cancel $\frac{x+11}{x+11}$

$$= \lim_{x \to -11^-} -5$$

no longer indeterminate

$$= -5$$

limit from the right

$$\lim_{x\to -11^+} \frac{5x+55}{|x+11|} = \lim_{x\to -11^+} \frac{5x+55}{x+11}$$

$\underbrace{x\to -11^+}_{x>-11}$

swap in the appropriate formula

$$= \lim_{x\to -11^+} \frac{5\cancel{(x+11)}}{\cancel{(x+11)}}$$

indeterminate

Since $x > -11$, we know $x+11 \neq 0$, so we can cancel $\frac{x+11}{x+11}$

$$= \lim_{x\to -11^+} 5$$

no longer indeterminate

$$= 5$$

$$\lim_{x \to -11} \frac{5x+55}{|x+11|}$$ does not exist

because left + right limits don't match.

Meeting Part 3

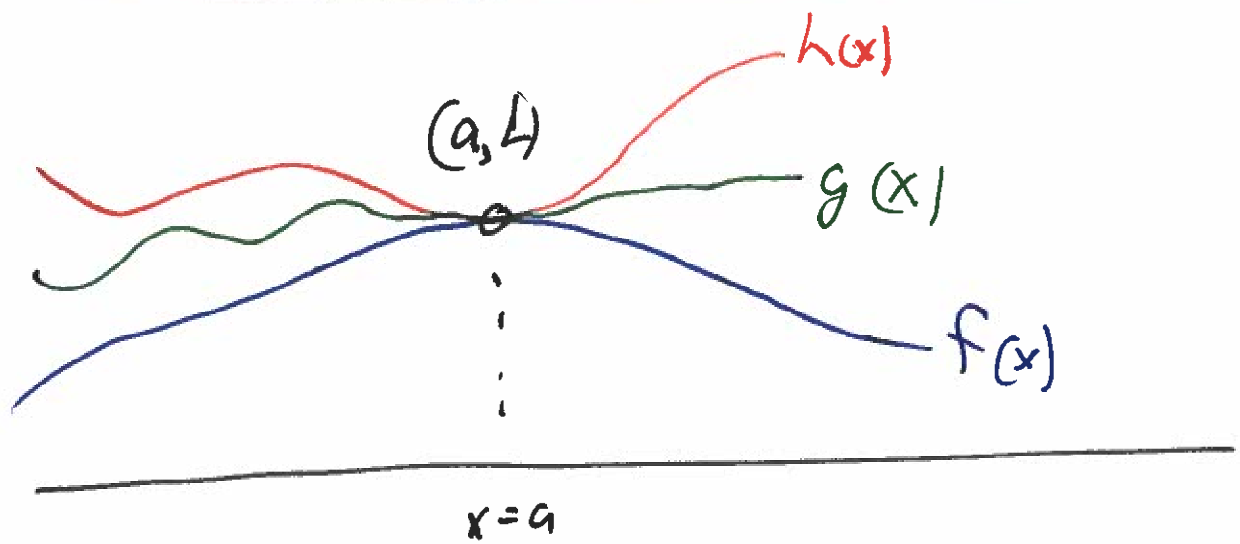
The Squeeze theorem

If $f(x) \leq g(x) \leq h(x)$

~~on~~ for all ~~x~~ x except possibly at $x=a$

and if $\lim_{x \to a} f(x) = L = \lim_{x \to a} h(x)$

then $\lim_{x \to a} g(x) = L$



example similar to 1.4 #3

Suppose $-x^2+3x-28 \leq g(x) \leq -3x \mp 19$ for all x

find $\lim\limits_{x \to 3} g(x)$

Solution

notice $\lim\limits_{x \to 3} -x^2+3x-28 \overset{\text{direct sub}}{=} -(3)^2+3(3)-28$

$= -9+9-28$

$= -28$

$\lim\limits_{x \to 3} -3x+19 \underset{\text{direct sub}}{=} -3(3) \mp 19 = -9 \mp 19$

$= -28$

we have satisfied all requirements of Squeeze theorem

So $\lim_{x \to 3} g(x) = -28$