MATH 2301 Section 110 (Barsamian) Meeting #40 (Mon Apr 3)

- Exam X3 This Friday April 7
  - Study Guide on Course Web Page
  - Notice: Not covering Section 4.6 Newton's Method this semester
- Web Page now has ~~___~~ links to lecture notes. (But keep taking notes!)

Meeting Part 1 Finish curvesketching example from Friday

Goal: Use Graphing Strategy to sketch graph of $f(x) = xe^{(x)}$

From Friday

We did Step 1 & Step 2 of Graphing Strategy

Finish Step 3 Sign chart for $f''(x) = (2+x)e^x$

$$\underset{}{- - - - - - -} \quad \underset{x=-2}{f''(-2)=0} \quad + + + + + \longrightarrow x$$

f concave down on $(-\infty, -2)$ because f'' is negative there

f concave up on $(-2, \infty)$ because f'' is positive there.

So f has inflection point at $x = -2$ because that x value is in the domain and the concavity changes there.

(2)

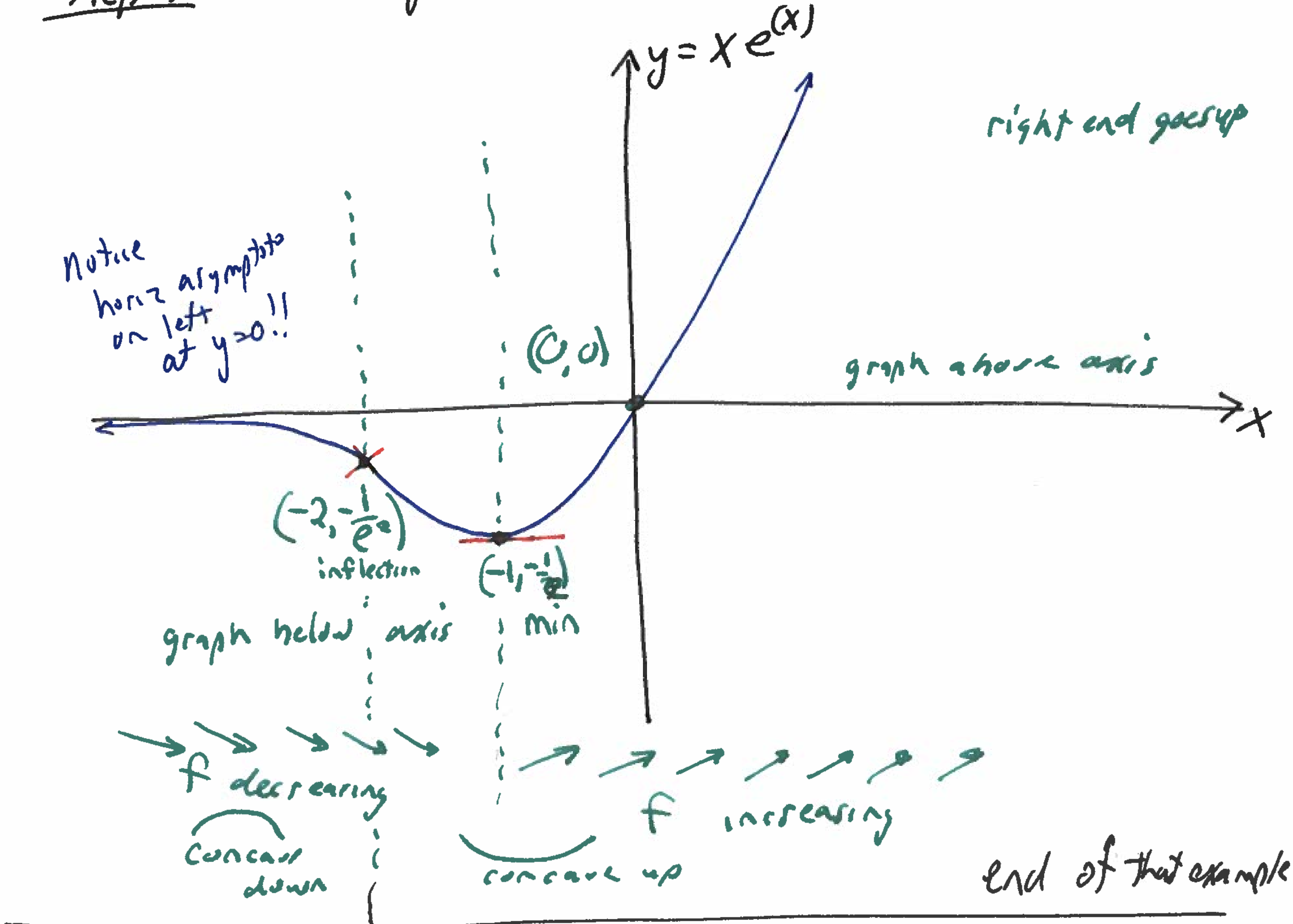
y coordinate of ~~the~~ inflection point will be

$$f(-2) = (-2)e^{(-2)} = -2 \cdot \frac{1}{e^2} = -\frac{2}{e^2}$$

So inflection point has coordinates

$$(x, y) = \left(-2, -\frac{2}{e^2}\right)$$

Step 4 Sketch graph



end of that example

Meeting Part 2 Section 4.5 Optimization

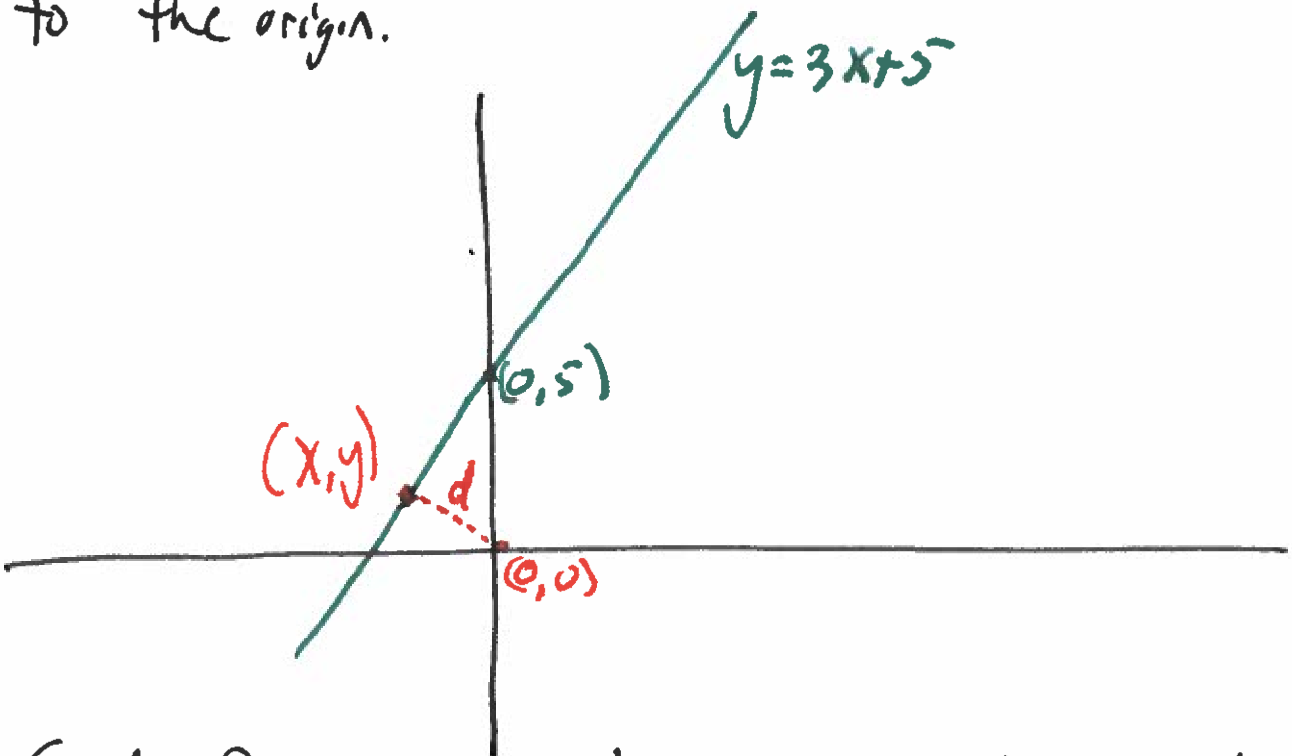
Optimization Problems are just max/min problems!

But there may be complications.

- often presented as word problems.
- often the functions are not given. you have to figure them out.
- Sometimes there are many variables. you will have to work to eliminate ~~to~~ all but one or two.

Example #1 (Similar to 4.5 #15)

Find point on the line $y = 3x+5$ that is closest to the origin.



Goal: find point (x,y) that minimizes d.

d = distance from point (x,y) to the origin

$= \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2+y^2}$ ugly formula!

Simpler plan

observe $d^2 = x^2 + y^2$

Find (x,y) ~~that~~ such that d^2 is minimized

$$\begin{cases} d^2 = x^2 + y^2 & \text{want to minimize } d^2 \\ y = 3x + 5 \end{cases}$$

use the second equation to eliminate y.

Substitute $y = 3x+5$ into 1st equation

$$d^2 = x^2 + (3x+5)^2 = x^2 + 9x^2 + 30x + 25$$

$$d^2 = 10x^2 + 30x + 25$$

Goal: Find value of x that minimizes d^2

Strategy find $(d^2)'$

set $(d^2)' = 0$

solve for x

$$(d^2)' = \frac{d}{dx}(d^2) = \frac{d}{dx}\left(10x^2+30x+25\right) = 20x+30$$

$$0 = 20x+30$$

$$-30 = 20x$$

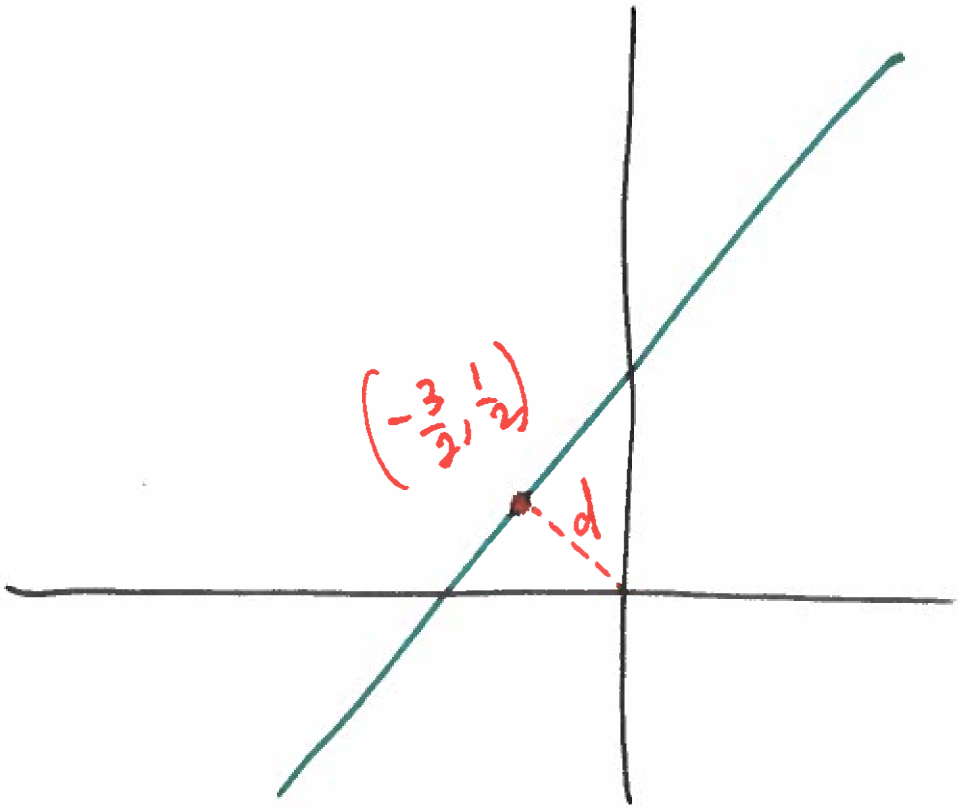
$$-\frac{30}{20} = x$$

$$x = -\frac{3}{2}$$

Corresponding value of y is

$$y = 3\left(-\frac{3}{2}\right)+5 = -\frac{9}{2}+5 = \frac{1}{2}$$

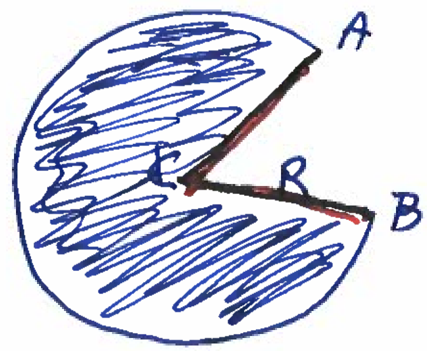
So $(x,y) = \left(-\frac{3}{2}, \frac{1}{2}\right)$



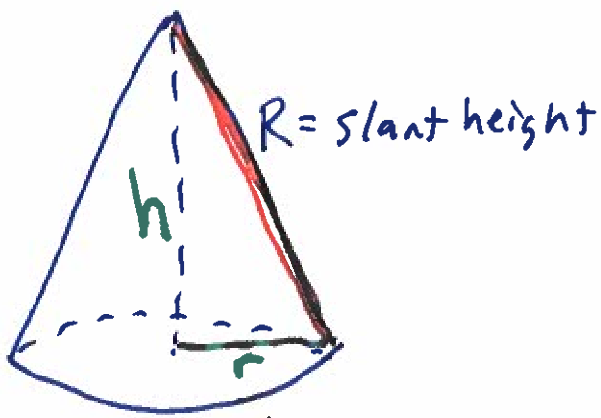
end of example

Example #2 4.5 #29

Cone shaped drinking cup made from circular piece of paper



Find max possible volume of such a cup.



$$V_{cone} = \frac{1}{3}(\text{Area of base})\cdot\text{height} = \frac{1}{3}\cdot\pi\cdot r^2\cdot h$$

we also know $r^2 + h^2 = R^2$

variables (r, h) — fixed constant (R)

use 2^{nd} equation to eliminate r^2

$$r^2 + h^2 = R^2$$

$$r^2 = R^2 - h^2$$

Substitute into first equation

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\cdot\pi(R^2 - h^2)\cdot h$$

Goal: maximize V.

- find h that maximizes V
- find resulting value of V

$$V = \frac{1}{3}\pi(R^2 - h^2) \cdot h$$

$$= \frac{1}{3}\pi(R^2h - h^3)$$

Find V'

$$V' = \frac{dV}{dh} = \frac{1}{3}\pi(R^2(1) - 3h^2)$$

$$V' = \frac{1}{3}\pi(R^2 - 3h^2)$$

Set $V' = 0$ and solve for h

$$0 = \frac{1}{3}\pi(R - 3h^2)$$

$$0 = R - 3h^2$$

$$3h^2 = R^2$$

$$h^2 = R^2/3$$

$$h = \sqrt{R^2/3} = \frac{R}{\sqrt{3}}$$

$$\text{So } V = \frac{1}{3}\pi(R^2 - h^2)h$$

$$= \frac{1}{3}\pi\left(R^2 - \left(\frac{R}{\sqrt{3}}\right)^2\right)\frac{R}{\sqrt{3}}$$

$$= \frac{1}{3}\pi\left(R^2 - \frac{R^2}{3}\right)\frac{R}{\sqrt{3}}$$

$$= \frac{1}{3}\pi R^2\left(1 - \frac{1}{3}\right)\frac{R}{\sqrt{3}}$$

$$= \frac{1}{3}\pi R^2\left(\frac{2}{3}\right)\frac{R}{\sqrt{3}}$$

$$V = \frac{2\pi R^3}{9\sqrt{3}}$$

end of example

End of Class Meeting #40