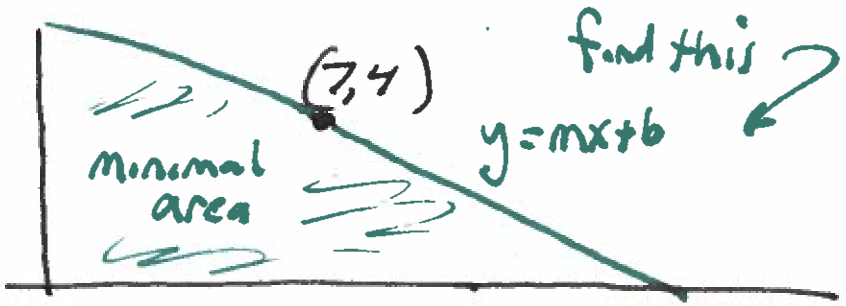
MATH 2301 Section 110 (Barsamian) Day 42 (Wed Apr 5)

- Remember that Lecture Notes are now available on the web page (in the calendar)
- Remember that Exam X3 is this Friday, Apr 7 (see study guide on web page, in the calendar)
- Remember that Section 4.6 Newton's Method has been removed from the course. (It won't be on Final)

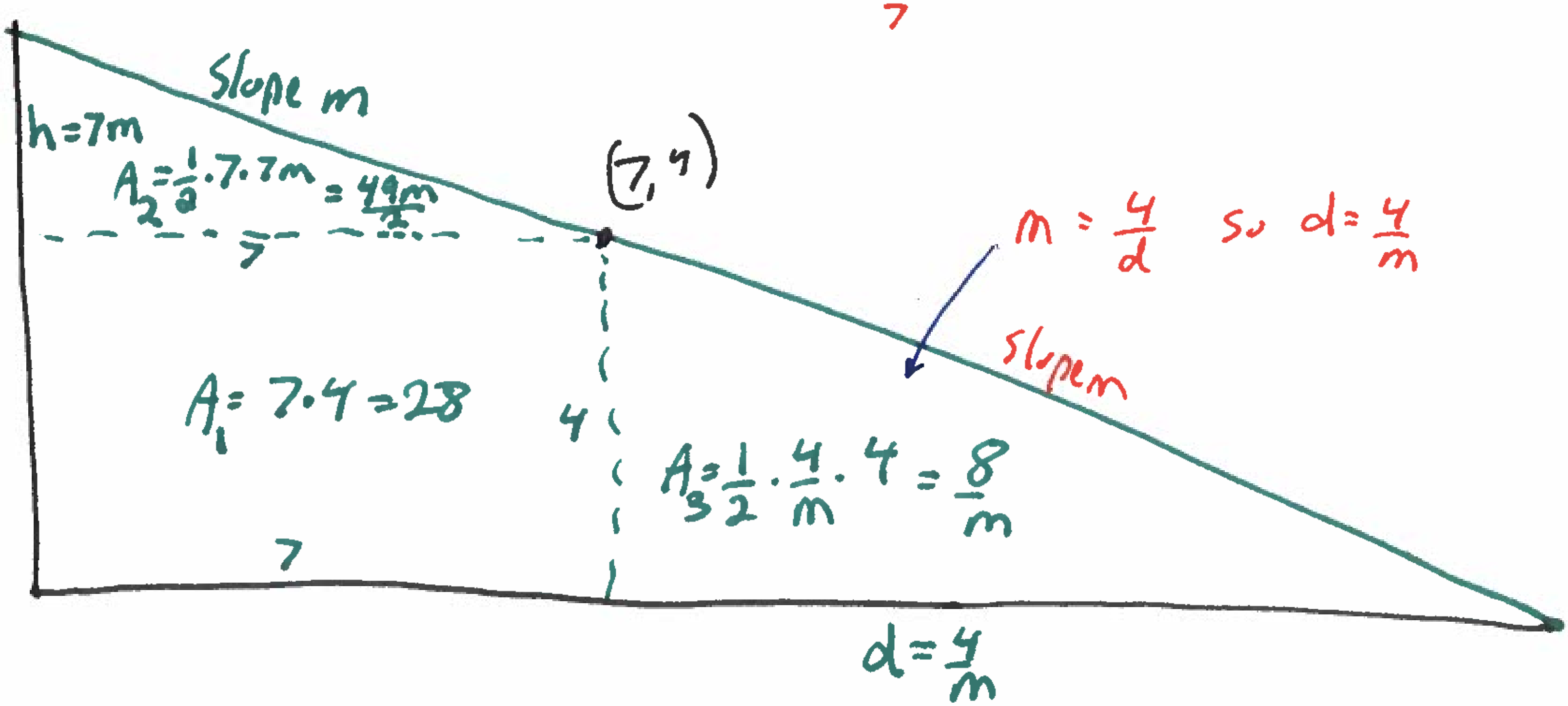
Meeting Part 1 One more optimization example

example similar to 4.5#39 Find an equation for the line through the point (7,4) that cuts off the least area from the first quadrant



Strategy find slope m that minimizes area. Then use that to find b.

$m = \frac{h}{7}$ so $h = 7m$



So total area is $A = A_1 + A_2 + A_3$

$$A(m) = 28 + \frac{49m}{2} + \frac{8}{m}$$

Find value of m that ~~min~~ minimizes A(m)

Strategy Find $A'(m)$
Set $A'(m) = 0$
Solve for m

~~$A'(m) = \frac{d}{dm}\left(28 + \frac{49m}{2} + \frac{8}{m}\right)$~~

First put $A(m)$ in power function form

$$A(m) = 28 + \frac{49m}{2} + \frac{8}{m} = 28 + \left(\frac{49}{2}\right)m + 8\cdot m^{-1}$$

then

$$A'(m) = \frac{d}{dm}28 + \left(\frac{49}{2}\right)\frac{dm}{dm} + 8\frac{d}{dm}m^{-1}$$

$$= 0 + \left(\frac{49}{2}\right)(1) + 8(-1m^{-1-1})$$

$$= \frac{49}{2} - \frac{8}{m^2}$$

$$0 = \frac{49}{2} - \frac{8}{m^2}$$

$$\frac{8}{m^2} = \frac{49}{2}$$

$$\frac{8 \cdot 2}{49} = m^2$$

$$\frac{16}{49} = m^2$$

$$m = \frac{4}{7} \quad \text{or} \quad \boxed{m = -\frac{4}{7}}$$

need negative m in order for the line to cut off area.

Now find b

We know

- line passes through (7,4)
- line has slope $m = -\frac{4}{7}$

So point slope form for the line equation is

$$(y-4) = \left(-\frac{4}{7}\right)(x-7)$$

Solve for y

$$y-4 = \left(-\frac{4}{7}\right)x + 4$$

$$\boxed{y = \left(-\frac{4}{7}\right)x + 8}$$

[End of Example]

Meeting Part 2

Section 4.7 Antiderivatives

Definition

Words: $\underset{\substack{\uparrow\\ \text{capital}}}{F(x)}$ is an antiderivative of $\underset{\substack{\uparrow\\ \text{lower case}}}{f(x)}$

meaning: $\underset{\substack{\uparrow\\ \text{lower}\\ \text{case}}}{f(x)}$ is the derivative of $\underset{\substack{\uparrow\\ \text{upper}\\ \text{case}}}{F(x)}$

$$\underset{\text{capital}}{F'(x)} = \underset{\text{lower case}}{f(x)}$$

$$\underset{\text{capital}}{F(x)} \xrightarrow{\text{take deriv}} \underset{\substack{\text{lower}\\ \text{case}}}{f(x)}$$

Example #1

Is $F(x) = \frac{x^3}{3} + 5$ an antideriv of $f(x) = x^2$?

Check

$$F'(x) = \frac{d}{dx}\left(\frac{x^3}{3} + 5\right) = \frac{1}{3}\frac{d}{dx}x^3 + \frac{d}{dx}5 = \frac{1}{3}(3x^2) + 0 = x^2 = f(x)$$

yes!

Example #2

Is ~~F~~ $G(x) = \frac{x^3}{3} + 17$ an antideriv of $f(x) = x^2$?

yes Check $G'(x) = \frac{d}{dx}\left(\frac{x^3}{3} + 17\right) = \dots = x^2 = f(x)$ ✓

Particular antiderivatives of $f(x) = x^2$

$$\frac{x^3}{3} + 5$$

$$\frac{x^3}{3} + 17$$

$$\frac{x^3}{3}$$

Most General Antiderivative. (General antideriv)

$$\frac{x^3}{3} + \underset{\substack{\uparrow \\ \text{unknown constant}}}{K}$$

We will build, bit by bit, a collection of <u>known</u> particular antiderivatives

The power rule for antiderivatives

function	known particular antiderivative
x^n with $n \neq -1$	$\frac{x^{n+1}}{n+1}$

If $\underset{\text{lower case}}{f(x)} = x^n$ then a known particular antideriv is $\underset{\text{upper case}}{F(x)} = \frac{x^{n+1}}{n+1}$

Why? Check

$$F'(x) = \frac{d}{dx}\left(\frac{x^{n+1}}{n+1}\right) = \frac{1}{(n+1)}\frac{d}{dx}x^{n+1} \underset{\substack{\uparrow \\ \text{power rule} \\ \text{for derivatives}}}{=} \cancel{\frac{1}{n+1}}\left(\cancel{(n+1)}\,x^{(n+1)-1}\right) = x^n = f(x)$$

Examples for each function find a particular antiderivative

[1] $f(x) = x^2$

$$F(x) = \cancel{\frac{x^{3+1}}{3}} \; \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

[2] $f(x) = \frac{1}{x^2} = x^{-2}$

↑ convert to power function form

$$F(x) = \frac{x^{-2+1}}{-2+1} = \frac{x^{-1}}{-1} = -x^{-1} = -\frac{1}{x}$$

check $F'(x) = \frac{d}{dx}\left(-\frac{1}{x}\right) = \cdots = \frac{1}{x^2} = f(x)$

[3] $f(x) = X \underset{\uparrow}{=} X^1$

↑ convert to power function form

$$F(x) = \frac{X^{1+1}}{1+1} = \frac{X^2}{2}$$

[4] $f(x) = 1 \underset{\uparrow}{=} X^0$

↑ convert to power function form

$$F(x) = \frac{X^{0+1}}{0+1} = \frac{X^1}{1} = X$$

Check $F'(X) = \frac{d}{dx}X = 1 = f(x)$ ✓

~~[5] $f(x) = \frac{1}{x} = x^{-1}$ ← convert to power function form~~

~~use power rule~~

~~$F(x) = \frac{x^{-1+1}}{-1+1} = \frac{x^0}{0} = \frac{1}{0}$ Does not exist!?!?~~

Invalid computation

Can't use the power rule when $n = -1$

the $1/x$ rule for antiderivatives

function	known particular antideriv
$\frac{1}{x}$	$\ln(\lvert x \rvert)$

Sum and constant multiple rule

If $f(x)$ and $g(x)$ are functions with known particular antiderivatives $F(x)$ and $G(x)$

lower case

capital

and a, b are constants,

then the function

$$af(x) + bg(x)$$

has known antideriv $aF(x) + bG(x)$

Example Similar to 4.7 #2

$$f(x) = \frac{2}{3} - 5X + \frac{7}{11}X^3 + \frac{13}{17X^5}$$

Find a particular antiderivative of f(x)

Solution Put f(x) in power function form

$$f(x) = \left(\frac{2}{3}\right)X^0 - 5X^1 + \left(\frac{7}{11}\right)X^3 + \left(\frac{13}{17}\right)X^{-5}$$

use power rule

$$F(x) = \left(\frac{2}{3}\right)\frac{X^{0+1}}{0+1} - 5\frac{X^{1+1}}{1+1} + \left(\frac{7}{11}\right)\frac{X^{3+1}}{3+1} + \left(\frac{13}{17}\right)\frac{X^{-5+1}}{-5+1}$$

$$= \left(\frac{2}{3}\right)X - 5\frac{X^2}{2} + \left(\frac{7}{11}\right)\frac{X^4}{4} + \left(\frac{13}{17}\right)\frac{X^{-4}}{-4}$$

$$= \frac{2X}{3} - \frac{5X^2}{2} + \frac{7X^4}{44} - \frac{13}{68X^4}$$

Example Similar to 4.7#13

$$f(x) = \frac{x^7 + 5x^3 + 3x^2}{x^4}$$

Find a particular antiderivative

Solution: Convert f(x) to power function and $\frac{1}{x}$ form

$$f(x) = \frac{x^7}{x^4} + \frac{5x^3}{x^4} + \frac{3x^2}{x^4}$$

$$= x^3 + \frac{5}{x} + \frac{3}{x^2}$$

$$= x^3 + 5\left(\frac{1}{x}\right) + 3x^{-2}$$

Particular Antideriv

$$F(x) = \frac{x^{3+1}}{3+1} + 5\ln(|x|) + 3\frac{x^{-2+1}}{-2+1}$$

capital

$$F(x) = \frac{x^4}{4} + 5\ln(|x|) + \frac{3x^{-1}}{-1}$$

$$= \frac{x^4}{4} + 5\ln(|x|) - 3x^{-1}$$

$$= \frac{x^4}{4} + 5\ln(|x|) - 3\left(\frac{1}{x}\right)$$

$$= \frac{x^4}{4} + 5\ln(|x|) - \frac{3}{x}$$

End of Example

End of Meeting