MATH 2301 Section 110 (Barsamian) Meeting #46 (Wed Apr 12)

Today: Section 5.2 The Definite Integral

Friday: Section 5.3 Evaluating Definite Integrals
Quiz Q8 covering Sections 5.1, 5.2

Comments on Exam X3

[2] $f(x) = \ln(x^2 - x + 5/4)$ on interval $[-1, 1]$

continuous — closed interval

To find abs extrema, must use closed interval method.

important x values	corresponding y values	
$x = -1$ end	$\ln((-1)^2 - (-1) + 5/4) = \ln(13/4)$	max
$x = 1/2$ crit	$\ln\left(\left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right) + 5/4\right) = \ln(1) = 0$	min
$x = 1$ end	$\ln(1^2 - 1 + 5/4) = \ln(5/4)$	

To find critical numbers, investigate $f'(x)$

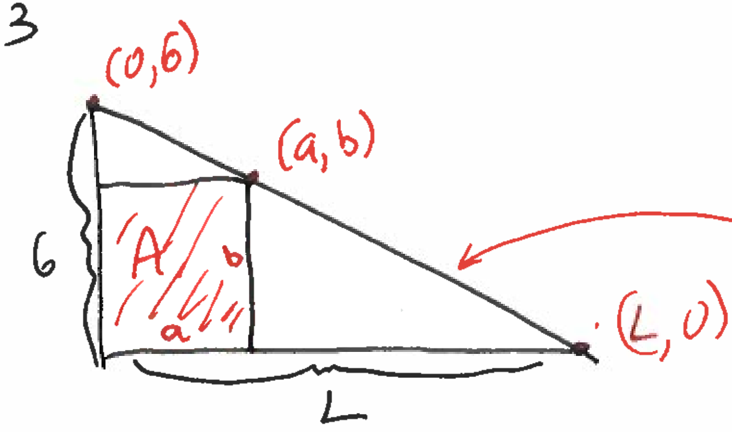
$$f'(x) \underset{\uparrow \text{ chain}}{=} \frac{2x-1}{x^2 - x + 5/4}$$

} numerator only zero when $x = 1/2$

} denominator will never be zero

Exam X3

[5]



slope $m = \frac{\Delta y}{\Delta x} = \frac{-6}{L}$

line equation

$$y = \left(-\frac{6}{L}\right)x + 6$$

So (a,b) must satisfy that equation

so $b = \left(-\frac{6}{L}\right)a + 6$

Maximize $A = a \cdot b = a\left(\left(-\frac{6}{L}\right)a + 6\right) = \left(-\frac{6}{L}\right)a^2 + 6a$

This is a downward facing parabola. Will have max where $A' = 0$

$$0 = A' = \frac{d}{da}\left(\left(\frac{-6}{L}\right)a^2 + 6a\right) = \left(\frac{-6}{L}\right)2a + 6$$

$$a = \frac{L}{2}$$

$$b = 3 \quad \Longrightarrow \quad A = \frac{3L}{2}$$

Monday we were talking Signed Area SA
between graph of f(x) and x axis from $x=a$ to $x=b$.

Definition of The Definite Integral

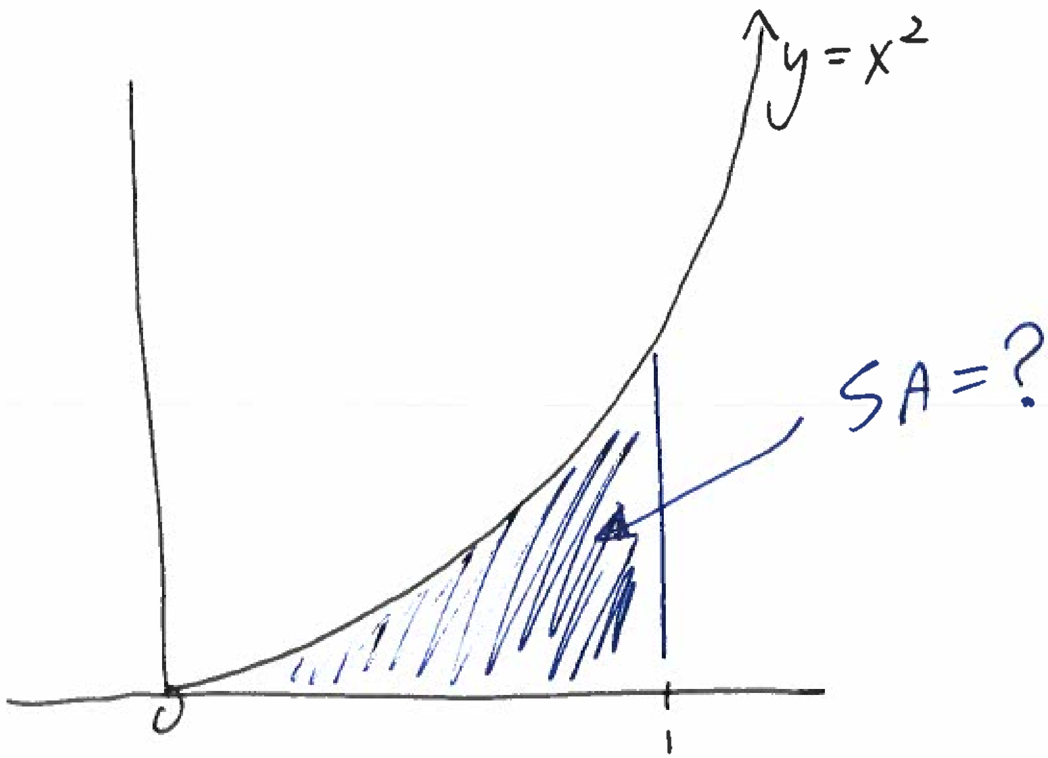
- Symbol: $\int_a^b f(x)\,dx$
- spoken: The definite integral of f(x) from $x=a$ to $x=b$.
- meaning: the number that is the following limit

$$\lim_{n\to\infty} L_n$$

the Left Riemann Sum with n rectangles

- Remark: When f is a continuous function, this limit always exists and equals $\lim_{n\to\infty} R_n$
- Additional terminology and notation: This definite integral can be denoted SA, and called the signed area between graph of f(x) and x axis from $x=a$ to $x=b$.

Example Find $\int_0^1 x^2 dx$ the signed area between graph of $f(x) = x^2$ and x axis from $x=0$ to $x=1$



On monday, we estimated $SA \approx 0.333$

Strategy
- Build expression for R_n ← Riemann sum with n right rectangles
- Find $\lim_{n\to\infty} R_n$

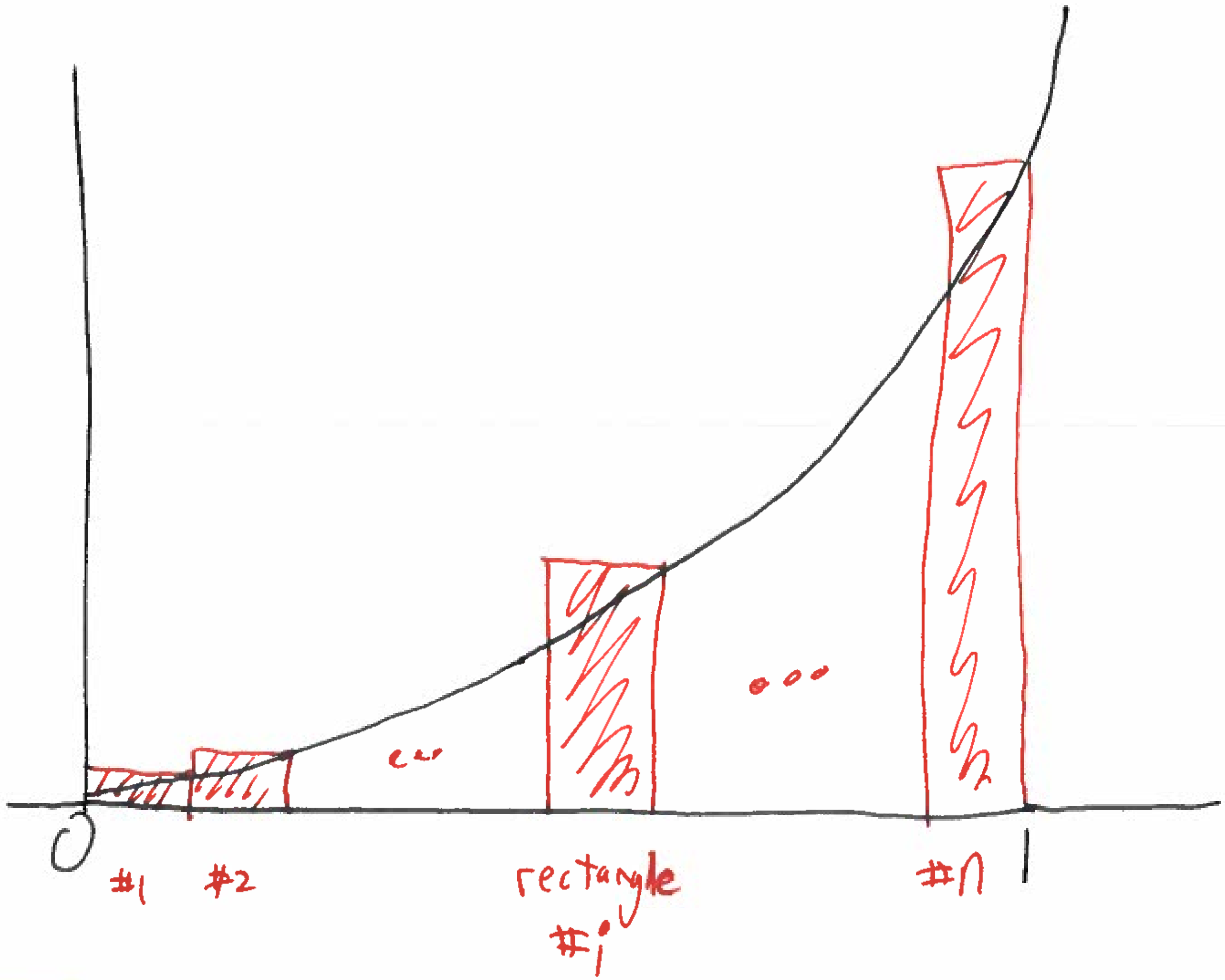


Figure out the <u>widths</u>

Rectangle widths are all the same

$$\Delta X = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$$

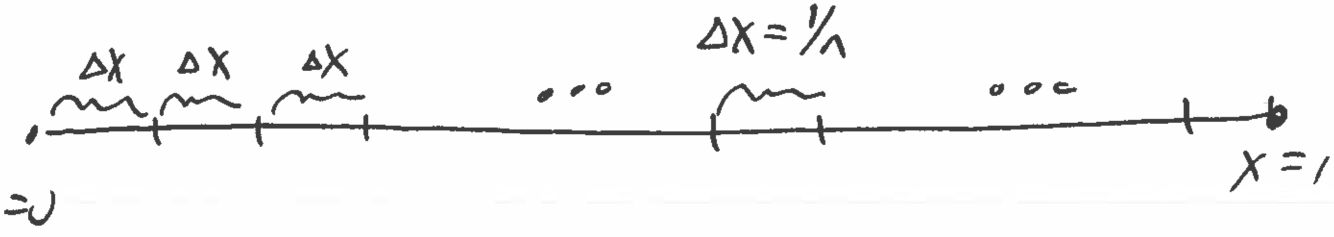
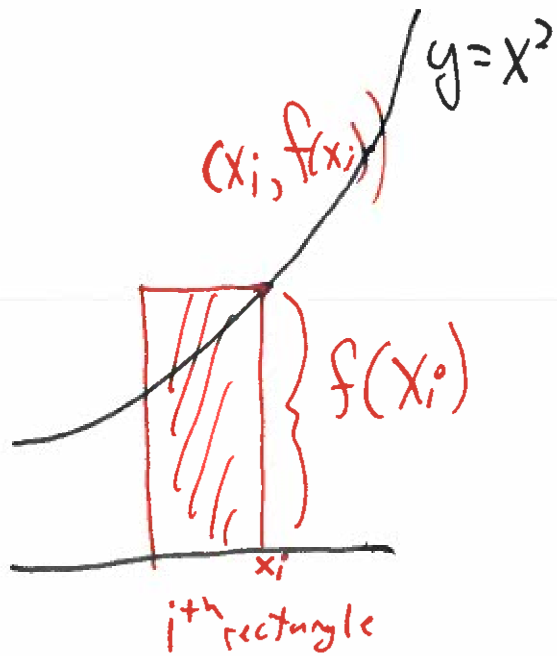


Figure out the <u>heights</u> of rectangles

Each Rectangle touches graph at rectangles right edge.

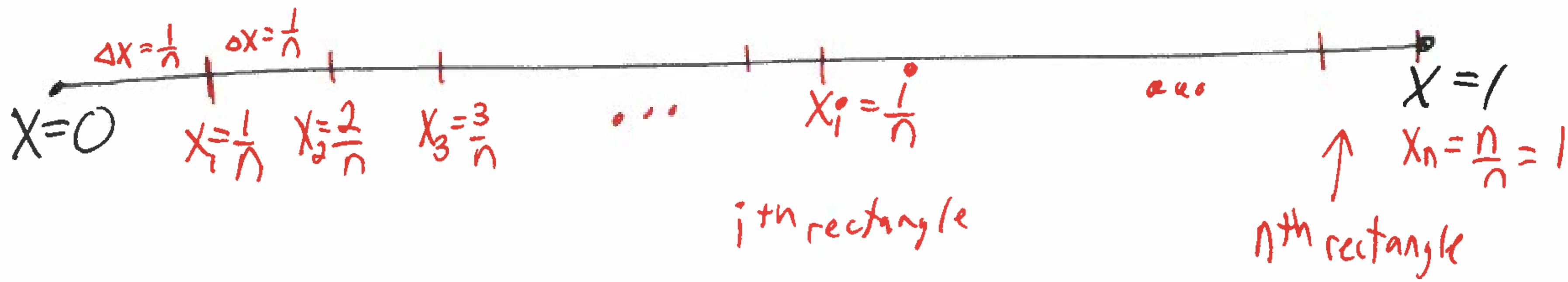
So height of rectangle = height of graph of $f(x)$ at right end of that little interval.



$= f(x_i)$

$= (x_i)^2$

We need to figure out the values of the x_i



So the heights of the rectangles are

$$\left(\frac{1}{n}\right)^2, \left(\frac{2}{n}\right)^2, \left(\frac{3}{n}\right)^2, \ldots, \left(\frac{i}{n}\right)^2, \ldots, \left(\frac{n}{n}\right)^2$$

So rectangle areas are height $\cdot$ width

$$\left(\frac{1}{n}\right)^2 \cdot \frac{1}{n}, \left(\frac{2}{n}\right)^2 \cdot \frac{1}{n}, \ldots, \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}, \ldots, \left(\frac{n}{n}\right)^2 \cdot \frac{1}{n}$$

$$\frac{1^2}{n^3}, \frac{2^2}{n^3}, \ldots, \frac{(i)^2}{n^3}, \ldots, \frac{n^2}{n^3}$$

Total area is the sum

$$R_n = \frac{1^2}{n^3} + \frac{2^2}{n^3} + \cdots + \frac{i^2}{n^3} + \cdots + \frac{n^2}{n^3}$$

$$= \frac{1}{n^3}\Big(\underbrace{1^2 + 2^2 + 3^2 + \cdots + i^2 + \cdots + n^2}_{\text{Sum of the squares of the first } n \text{ positive integers}}\Big)$$

$$= \frac{1}{\cancel{n^3}\,n^2} \cdot \frac{\cancel{n}(n+1)(2n+1)}{6}$$

$$= \frac{(n+1)(2n+1)}{6n^2}$$

Simplify to a form that is easier for the limit

$$= \frac{2n^2 + 3n + 1}{6n^2}$$

$$R_n = \frac{2n^2}{6n^2} + \frac{3n}{6n^2} + \frac{1}{6n^2}$$

$$R_n = \frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}$$

Now take the limit as $n \to \infty$ to find SA

$$SA = \lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(\frac{1}{3} + \frac{1}{2n} + \frac{1}{6n^2}\right)$$

$$= \lim_{n\to\infty}\left(\frac{1}{3}\right) + \lim_{n\to\infty}\left(\frac{1}{2n}\right) + \lim_{n\to\infty}\left(\frac{1}{6n^2}\right)$$

$\frac{1}{\text{huge number}}$ = small

$$= \frac{1}{3} + 0 + 0$$

Conclusion

$$SA = \int_0^1 x^2\,dx = \frac{1}{3}$$

Final answer

Notice that this answer agrees with our estimate on Monday, when we estimated that the signed area was ~~going to be~~ probably around 0.333

~~Question~~

Question for Friday:

Finding Definite Integrals by finding the limit of a Riemann Sum

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} R_n$$

Is <u>really</u> <u>hard</u>!!

Is there an <u>easier way</u> to find the value of a definite integral?!?

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end of meeting