MATH 2301 Section 110 (Barsamian) Meeting #47 (Fri Apr 14)

- Today: Discuss Section 5.3
  Quiz Q8
- Next Friday (April 21) Quiz Q9
- Tues May 2: Final exam 4:40 pm room to be announced later.

Section 5.3 Meeting Part 1 Indefinite Integrals

Indefinite Integrals

symbol: $\int f(x)dx$

usage: f is a continuous function

spoken: the indefinite integral of $f(x)$.

meaning: the general antiderivative of $f(x)$

Example for $f(x) = x^2$

we can find a particular antiderivative by using the power rule for antiderivatives

$$F(x) = \frac{x^{2+1}}{2+1} = \frac{x^3}{3}$$

Check $F'(x) = \frac{d}{dx}\left(\frac{x^3}{3}\right) = x^2 = f(x)$ ✓

there are many particular antiderivatives

$$\frac{x^3}{3}$$

$$\frac{x^3}{3} + 5$$

$$\frac{x^3}{3} - 17$$

$\xrightarrow{\text{take derivative}}$ $f(x) = x^2$

The "general antiderivative" is this function form

$$F(x) = \frac{x^3}{3} + C$$ where C is an unknown constant.

That is

$$\int x^2 \, dx = \frac{x^3}{3} + C$$

Power Rule for Indefinite Integrals

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$$ when $n \neq -1$

logarithm Rule For Indefinite Integrals

$$\int \frac{1}{x} \, dx = \ln(|x|) + c$$

Meeting Part 2

Evaluation notation

Symbol: $g(x)\Big|_{x=a}$

Spoken: "g evaluated at a"

means: $g(a)$

Example $(x^2+3x+2)\Big|_{x=5} = (5)^2+3(5)+2 = 42$

More evaluation notation

Symbol $g(x)\Big|_a^b$

Spoken: g evaluated from a to b.

Meaning: $g(b) - g(a)$

Example

$$\left(x^2+3x+2\right)\Big|_4^5 = \left((5)^2+3(5)+2\right) - \left(4^2+3(4)+2\right)$$

$$= 40 - 30$$

$$= 10$$

Meeting Part 3 Evaluating Definite Integrals

We Defined

$$\underset{\text{Signed area}}{SA} = \underbrace{\int_a^b f(x)\,dx}_{\text{Definite Integral}} \underset{\text{definition}}{=} \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$$

this is really hard.

Natural Question: Is there an easier way to find the Value of a Definite Integral?

Answer: yes!

The Evaluation theorem

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is any particular antiderivative of $f(x)$

using Evaluation Notation

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

The evaluation theorem written using indefinite integrals

$$\int_a^b f(x)\,dx \underset{\substack{\text{evaluation}\\\text{theorem}}}{=} \left(\int f(x)\,dx\right)\Bigg|_a^b$$

Example #1 Find the Definite Integral

$$\int_0^1 x^2\,dx \underset{\substack{\text{evaluation}\\ \text{theorem}}}{=} \left(\int x^2\,dx\right)\Big|_0^1$$

Indefinite Integral Details

$$\int x^2\,dx \underset{\substack{\uparrow\\ \text{power rule}\\ \text{with } n=2}}{=} \frac{x^{2+1}}{2+1} + C$$

$$= \frac{x^3}{3} + C$$

$$= \left(\frac{x^3}{3} + C\right)\Big|_0^1$$

$$= \left(\frac{(1)^3}{3} + \cancel{C}\right) - \left(\cancel{\frac{(0)^3}{3}}^{\,0} + \cancel{C}\right)$$

$$= \frac{1}{3}$$

Example Find the Definite Integral

$$\int_1^4 \frac{\sqrt{x}-x}{x^2}\,dx \underset{\text{F.T.}}{=} \left(\int \frac{\sqrt{x}-x}{x^2}\,dx\right)\Bigg|_1^4$$

Indefinite Integral Details

$$\int \frac{\sqrt{x}-x}{x^2}\,dx = \int \frac{\sqrt{x}}{x^2} - \frac{x}{x^2}\,dx$$

$$= \int x^{-3/2} - \frac{1}{x}\,dx$$

$$= \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} - \ln|x| + C$$

$$= \frac{x^{-1/2}}{-1/2} - \ln|x| + C$$

$$= -\frac{2}{\sqrt{x}} - \ln|x| + C$$

$$= \left(-\frac{2}{\sqrt{x}} - \ln|x| + C\right)\Bigg|_1^4$$

$$= \left(-\frac{2}{\sqrt{4}} - \ln|4| + \cancel{C}\right) - \left(-\frac{2}{\sqrt{1}} - \ln|1| + \cancel{C}\right)$$

Since 4 is positive, we can simplify $|4|$, replacing it with just 4.

$$= -\frac{2}{2} - \ln(4) + \frac{2}{1} + 0$$

$\ln|1| = \ln(1) = 0$

Since 1 is pos, we can replace $|1|$ with just 1

$$= 1 - \ln(4)$$