MATH 2301 Section 110 (Barsamian) Meeting #48 (Mon Apr 17, 2023)

Today

Section 5.3 The Evaluation theorem

Discuss a bit of Section 5.4 The Fundamental Theorem

Wed Section 5.4

Friday Section 5.4
Quiz Q9 (cover 5.3 + 5.4)

Next week Recitation, but NO QUIZ

Tues May 2 Combined Final Exam 4:40pm–6:40pm
room to be announced.

Continue Discussion of section 5.3

The Evaluation Theorem (discussed on Friday)

lowercase f   capital F

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left(\int f(x)\,dx\right)\Big|_a^b$$

Definite Integral

computation involving $F(x)$ (capital), an antiderivative of $f(x)$ (lower case)

Computation involving indefinite integral

Expresses the relationship between Definite Integrals and Indefinite Integrals

Meeting Part 1
Another Evaluation Theorem Example

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Example (5.3 #17) Evaluate $\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta$

Solution

$$\int_0^{\pi/4} \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta \underset{E.T.}{=} \left(\int \frac{1+\cos^2(\theta)}{\cos^2(\theta)}\,d\theta\right)\Big|_0^{\pi/4}$$

Indefinite Integral Details

$$\int \frac{1+\cos^2\theta}{\cos^2\theta}\,d\theta = \int \frac{1}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)}\,d\theta$$

$$= \int \sec^2(\theta) + 1\,d\theta$$

$$= \tan(\theta) + \theta + C$$

$$= \left(\tan(\theta) + \theta + C\right)\Big|_0^{\pi/4}$$

$$= \left(\tan(\pi/4) + \frac{\pi}{4} + \cancel{C}\right)$$

$$- \left(\cancel{\tan(0)}^{\,0} + \cancel{0} + \cancel{C}\right)$$

$$= 1 + \frac{\pi}{4}$$

$$\tan\left(\frac{\pi}{4}\right) = \frac{\sin(\pi/4)}{\cos(\pi/4)} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$$

Check: $\frac{d}{d\theta}\tan(\theta) = \frac{d}{d\theta}\frac{\sin(\theta)}{\cos(\theta)}$

$$= \frac{\left(\frac{d}{d\theta}\sin(\theta)\right)\cos(\theta) - \sin(\theta)\left(\frac{d}{d\theta}\cos(\theta)\right)}{(\cos(\theta))^2}$$

$$= \frac{(\cos(\theta))\cos(\theta) - \sin(\theta)(-\sin(\theta))}{\cos^2(\theta)}$$

$$= \frac{\cos^2(\theta) + \sin^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)}$$

$= \sec^2(\theta)$

Meeting Part 2 The Net Change theorem

The Evaluation theorem says $\int_a^b f(x)\,dx = F(b) - F(a)$

In this expression, $F(x)$ is an antiderivative of $f(x)$.

But this just means $F'(x) = f(x)$

use this symbol inside the integral, instead of $f(x)$

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

all capital F

Switch to $g(x)$

$$g(b) - g(a) = \int_a^b g'(x)\,dx$$

Switch to $f(x)$

$$\cancel{f(x)}\ f(b) - f(a) = \int_a^b f'(x)\,dx$$

The Net Change Theorem

$$\Delta f = f(b) - f(a) = \int_a^b f'(x)\,dx$$

basically, just a repackaging of the Evaluation Theorem.

One can find the change in a quantity by integrating the derivative of the quantity

Example 5.3 #60

An object moving in one dimension has velocity

$$v(t) = t^2 - 2t - 8 \quad \text{for } 1 \le t \le 6$$

(a) Find the ~~displacemen~~ displacement over that time interval

(b) Find the distance traveled

Solution

Review:

In physics problems involving motion,

t = variable representing time

$s(t)$ = position at time t

$v(t)$ = velocity at time t = $s'(t)$

Displacement from time a to time b is $\Delta s = s(b) - s(a)$

For our problem

In part (a)

$$\text{displacement} = \Delta S = S(6) - S(1) \underset{\substack{\text{Net}\\ \text{Change}\\ \text{Theorem}}}{=} \int_1^6 s'(t)\,dt$$

$$= \int_1^6 v(t)\,dt$$

because $s'(t) = v(t)$

$$= \int_1^6 t^2 - 2t - 8\,dt$$

$$= \left(\int t^2 - 2t - 8\,dt\right)\Bigg|_1^6$$

$$= \left(\frac{t^3}{3} - t^2 - \overset{8t}{\cancel{8}} + C\right)\Bigg|_1^6 =$$

Indefinite Integral Details

$$\int t^2 - 2t - 8\,dt = \frac{t^3}{3} - \frac{2t^2}{2} - 8t + C$$

$$= \frac{t^3}{3} - t^2 - \overset{8t}{\cancel{8}} + C$$

$$= \left(\frac{6^3}{3} - 6^2 - 8(6) + \cancel{C}\right) - \left(\frac{(1)^3}{3} - 1^2 - 8(1) + \cancel{C}\right)$$

$$= (72 - 36 - 48) - \left(\frac{1}{3} - 1 - 8\right)$$

$$= 36 - 48 - \frac{1}{3} + 9$$

$$= 45 - 48 - \frac{1}{3}$$

$$= -3 - \frac{1}{3}$$

$$= -\frac{10}{3}$$ displacement

(b) Distance traveled: will figure this out on <u>Wednesday</u>

End of Meeting