MATH 2301 Section 110 (Barsamian) meeting #50 (Wed Apr 19, 2023)

Today: Finish example about Net Change (Section 5.3)
Discuss the Fundamental Theorem of Calculus (Section 5.4)

Friday: Discuss Section 5.4 some more.
Quiz Q9 (over 5.3)

Tue May 2: Final Exam 4:40pm – 6:40pm

Resume Work on Example from ~~Wednesday~~ Monday

Exercise 5.3 #60

Object moving in One dimension with velocity in meters/second

$$v(t) = t^2 - 2t - 8 \quad \text{for} \quad 1 \le t \le 6 \quad \text{in seconds}$$

ⓐ Find displacement over that time interval.

Solution (from monday)

$$\text{displacement} = \underbrace{\Delta S}_{\text{change in position}} = S(6) - S(1) \underset{\text{NCT}}{=} \int_{t=1}^{t=6} S'(t)\,dt = \int_{t=1}^{t=6} v(t)\,dt$$

S(t) is the position function

Net Change theorem

because $v(t) = S'(t)$

$$= \int_1^6 t^2 - 2t - 8\,dt = \cdots\cdots = -\frac{10}{3} \text{ meters}$$

From Monday's notes

(b) Find <u>distance travelled</u> during that time interval

<u>Solution</u> we need to talk about speed & distance.

$$\text{speed}(t) = |v(t)|$$

$$\text{distance travelled} = \underset{\text{for distance}}{d} = \int_{t=a}^{t=b} \text{speed}(t)\,dt = \int_{t=a}^{t=b} |v(t)|\,dt$$

So for our example,

$$\text{distance travelled} = d = \int_{t=1}^{t=6} |v(t)|\,dt = \int_{t=1}^{t=6} |t^2-2t-8|\,dt$$

We can't integrate this with the absolute value symbol there. We have to figure out how to write $|t^2-2t-8|$ <u>without</u> absolute values.

Recall definition of the absolute value function

$$|x| = \begin{cases} x & \text{when } x \geq 0 \\ -x & \text{when } x < 0 \end{cases}$$

Piecewise-defined function

More clearly

when $x \geq 0$ the symbol $|x|$ ~~mean~~ means x.

when $x < 0$ the symbol $|x|$ means $-x$.

That is,

If $x \geq 0$ ~~the compute~~ use formula $|x| = x$ to get $|x|$.

If $x < 0$ use the formula $|x| = -x$ to get ~~the~~ $|x|$

Example

Let $X=17$ Find $|X|$

~~PE~~ $|17| = 17$

↑ use formula $|X| = X$ because $X \geq 0$

Let $X=-5$ Find $|X|$

$|-5| = -(-5) = 5$

↑ since $-5<0$, we use formula $|X| = -X$

More generally, to compute $|BLAH|$

If $BLAH \geq 0$ then the formula for $|BLAH|$ is

$$|BLAH| = BLAH$$

If $BLAH < 0$ then ~~the formula~~ $|BLAH|$

means $-BLAH$

So to figure out $|v(t)| = |t^2-2t-8|$

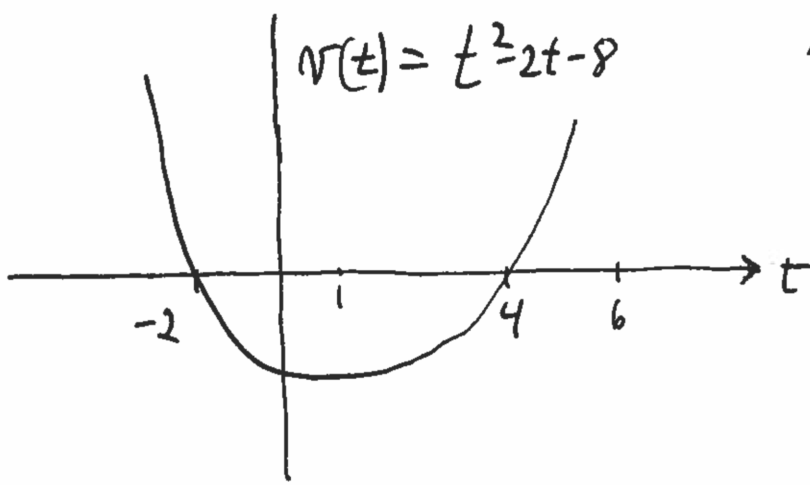
we have to figure out the sign of $v(t)$ on its domain

$v(t) = t^2-2t-8 \qquad = \qquad (t+2)(t-4)$

parabola facing up
Standard form

factored form

horizontal axis intercepts are at $t=-2$ and $t=4$



On our time interval $1 \le t \le 6$

⑧

So speed $= |v(t)| = \begin{cases} t^2-2t-8 & \text{when } t \ge 4 \\ -(t^2-2t-8) & \text{when } 1 \le t < 4 \end{cases}$

to integrate $\int_{t=1}^{t=6} |v(t)|\,dt$ we have to break up the integral into two chunks

distance $d_{[1,6]} = d_{[1,4]} + d_{[4,6]}$

distance from $t=1$ to $t=4$ distance from $t=4$ to $t=6$

$$= \int_{t=1}^{t=4} \underbrace{-t^2+2t+8}_{-v(t)}\,dt + \int_{t=4}^{t=6} \underbrace{t^2-2t-8}_{v(t)}\,dt$$

Do these two integrals separately

$$d_{[1,4]} = \int_{t=1}^{t=4} -v(t)\,dt = \int_{t=1}^{t=4} -t^2+2t+8\,dt$$

$$\stackrel{FTC}{=} \left(\int -t^2 + 2t + 8\,dt\right)\Bigg|_{t=1}^{t=4}$$

Indefinite Integral

$$\int -t^2+2t+8\,dt = -\frac{t^3}{3}+t^2+8t+C$$

$$= \left(-\frac{t^3}{3}+t^2+8t+C\right)\Bigg|_{t=1}^{4}$$

$$= \left(-\frac{(4)^3}{3}+(4)^2+8(4)+\cancel{C}\right) - \left(-\frac{(1)^3}{3}+(1)^2+8(1)+\cancel{C}\right)$$

$$= -\frac{64}{3}+16+32+\frac{1}{3}-1-8$$

$$= -\frac{63}{3}+39 \qquad = -21+39 = 18$$

$$d_{[4,6]} = \int_{t=4}^{t=6} v(t)\,dt\,\cancel{dt} = \int_{t=4}^{t=6} t^2 - 2t - 8\,dt$$

$$\underset{FTC}{=} \left(\int t^2 - 2t - 8\,dt\right)\Big|_4^6$$

$$= \left(\frac{t^3}{3} - t^2 - 8t + C\right)\Big|_4^6$$

$$= \left(\frac{(6)^3}{3} - (6)^2 - 8(6) + \cancel{C}\right) - \left(\frac{(4)^3}{3} - (4)^2 - 8(4) + \cancel{C}\right)$$

$$= \frac{216}{3} - 36 - 48 - \frac{64}{3} + 16 + 32$$

$$= 72 - 36 \cancel{- 48} - \frac{64}{3} \cancel{+ 16 + 32}$$

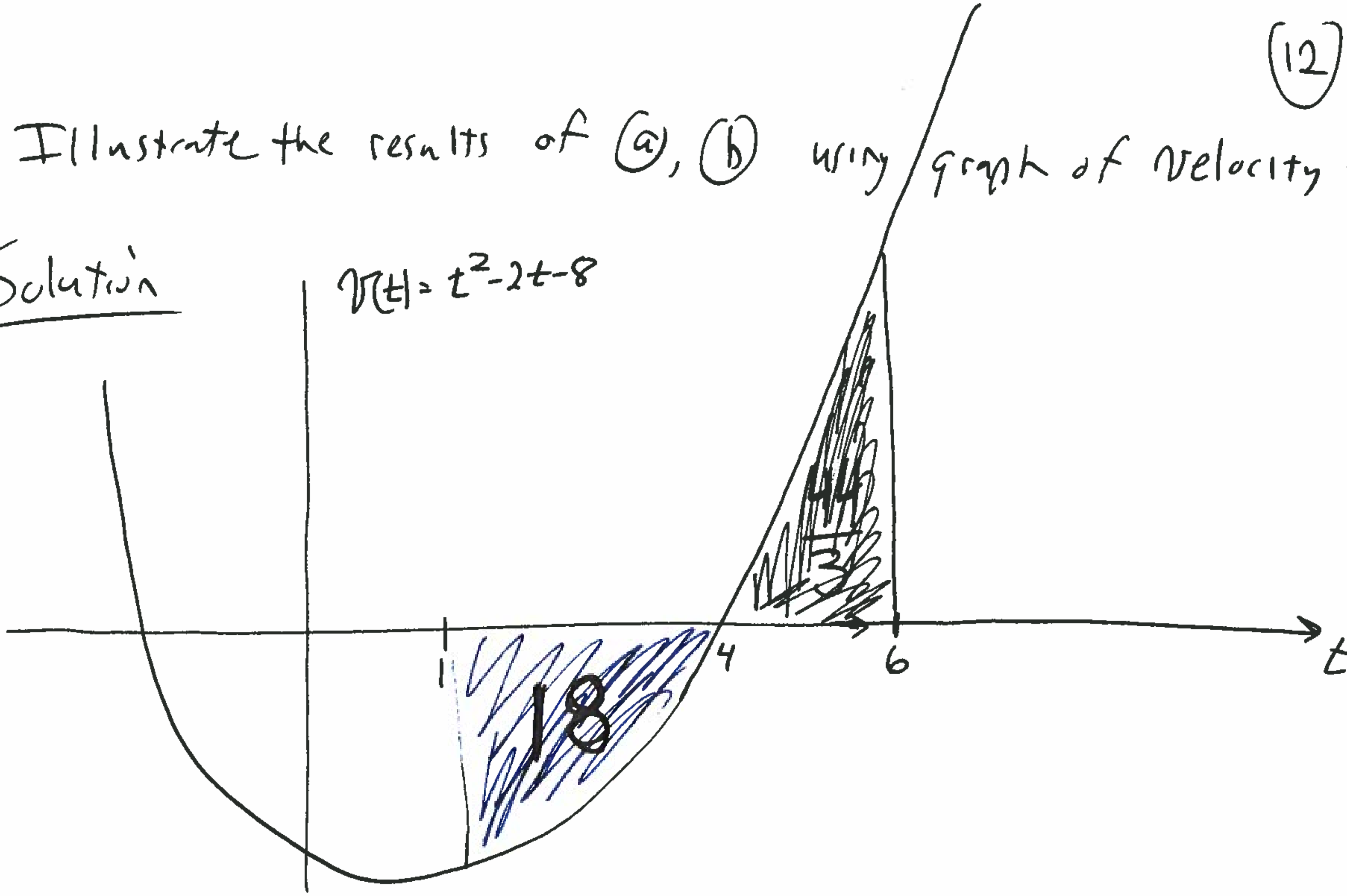
$$= 36 - \frac{64}{3} = \frac{108}{3} - \frac{64}{3} = \frac{44}{3}$$

So the total distance travelled is

$$d_{[1,6]} = d_{[1,4]} + d_{[4,6]} = 18 + \frac{44}{3} = \frac{98}{3} \text{ meters}$$

(c) Illustrate the results of (a), (b) using graph of velocity $v(t)$

Solution



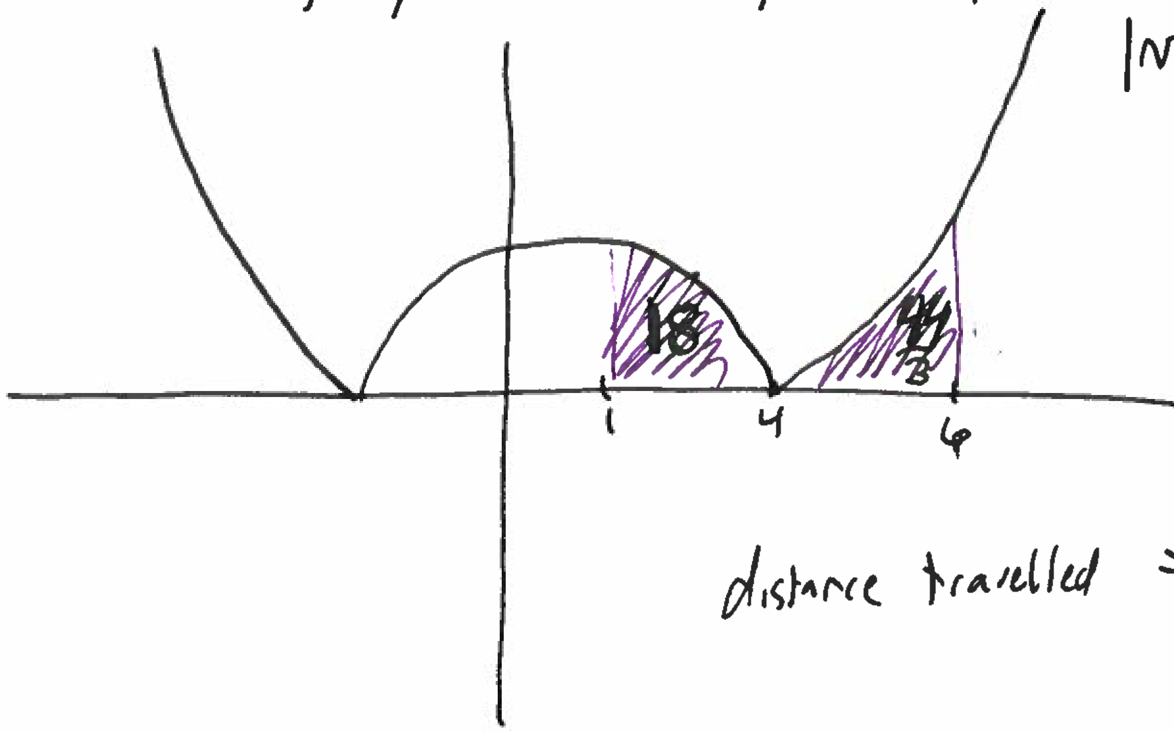
monday: $\Delta S = \int_1^6 v(t)\,dt$ = Signed area $= -18 + \frac{44}{3} = \frac{-10}{3}$
SA

today: distance $d_{[1,6]} = \int_1^6 |v(t)|\,dt$ = unsigned area $= 18 + \frac{44}{3} = \frac{98}{3}$
USA

End of (c)

The book would illustrate distance calculation by making graph of $|v(t)|$

$|v(t)| = |t^2 - t^2 - 8|$



distance travelled $= 18 + \frac{44}{3} = \frac{98}{3}$

End of Meeting