MATH 2301 Section 110 (Barsamian) Meeting #51 (Fri Apr 21) ①

Today Section 5.4 Fundamental Theorem of Calculus

Quiz Q9

Tues May 2, Final Exam 4:40 - 6:40

Recall two things

Thing #1

Signed Area between Graph of $f(x)$ and x axis from $x=a$ to $x=b$ $= SA = {}_aSA_b = A_{[a,b]} = \int_a^b f(x)\,dx$

A variety of symbols could be invented to denote Signed Area from a to b

Thing #2 The Evaluation theorem

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

lower case f capital F

The thing on the outside F, is an antiderivative of the function inside, $f(x)$

The thing on the inside, f, is the derivative of the function outside, F,

$$F'(x) = f(x)$$

Rewrite the evaluation theorem in different ways

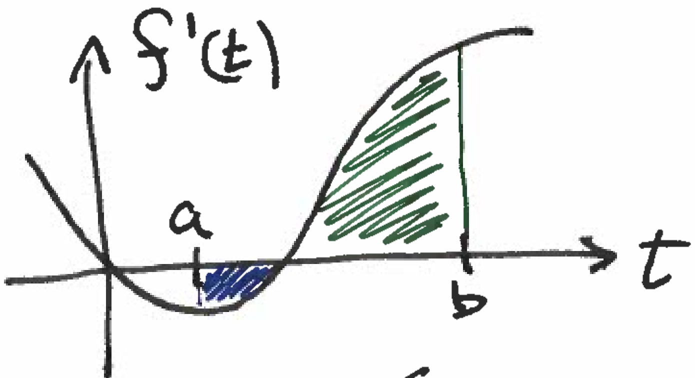
$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

$$\int_a^b g'(x)\,dx = g(b) - g(a)$$

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

We can use a different letter inside

$$\int_a^b f'(t)\,dt = f(b) - f(a) \quad \text{The Evaluation Theorem}$$



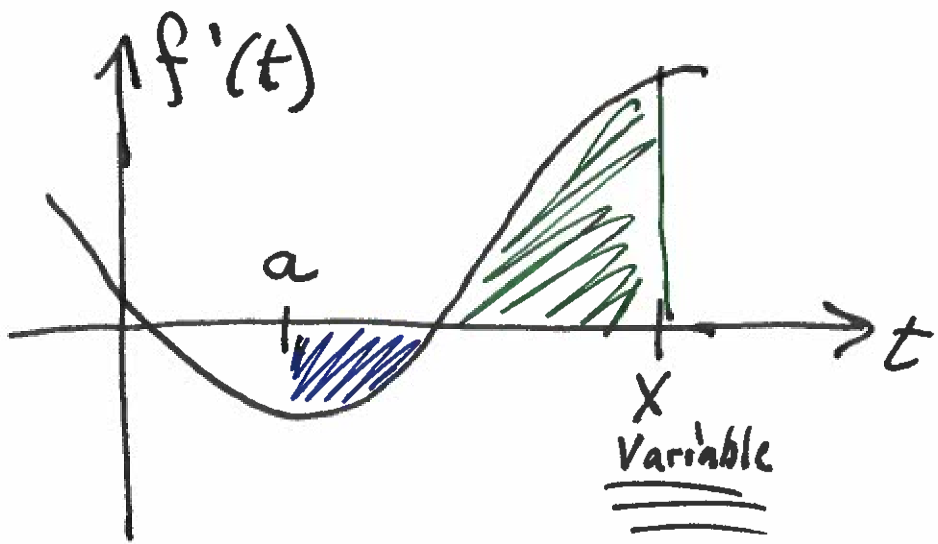
$$= \underbrace{f(b) - f(a)}_{\text{a number}}$$

evaluation theorem

SA

a number

$$\int_a^x f'(t)\,dt$$



$$= f(x) - f(a)$$

evaluation theorem

this is a function

This signed area is a function of the variable x

denoted $A_a(x)$

The Area function starting at a

⑤

$$ {}_aA(x) = \int_a^x f'(t)\,dt = f(x) - f(a) $$

what would be the ~~the~~ derivative $\frac{d}{dx}$ of all this stuff?

$$ \frac{d}{dx}\left({}_aA(x)\right) = \frac{d}{dx}\left(\int_a^x f'(t)\,dt\right) = \frac{d}{dx}\left(f(x) - f(a)\right) $$

$$ = \frac{d}{dx}f(x) - \cancel{\frac{d}{dx}\underbrace{f(a)}_{\text{a constant}}} \to 0 $$

$$ = f'(x) $$

Conclusion

$$\frac{d}{dx}\left(\int_a^x f'(t)\,dt\right) = f'(x)$$

There is nothing special about f. We could use g instead

$$\frac{d}{dx}\left(\int_a^x g'(t)\,dt\right) = g'(x)$$

We don't even have to keep the prime. The key thing is that the integrand gets used to build the result

$$\boxed{\frac{d}{dx}\left(\int_a^x f(t)\,dt\right) = f(x)}$$

This Result is famous and extremely important. It is called The Fundamental Theorem of Calculus, Part 1

Our earlier theorem, the "Evaluation Theorem", is also extremely important. It gets bundled in ~~[illegible]~~ to become part of the Fundamental theorem, too

$$\boxed{\int_a^b f(x)\,dx = \left(\int f(x)\,dx\right)\Big|_a^b}$$

Fundamental Theorem of Calculus Part 2

Example #1

Let $g(x) = \int_0^x 2 + \sin(t)\,dt$

Find $\frac{d}{dx} g(x)$

Solution

$$\frac{d}{dx} g(x) = \frac{d}{dx}\left(\int_0^x 2 + \sin(t)\,dt\right) = 2 + \sin(x)$$

End of Example #1

$\boxed{\text{Example\#2}}$ Let $y = \int_0^x \sqrt{t + \sqrt{t}}\, dt$

Find $\frac{dy}{dx}$

Solution $\frac{dy}{dx} = \frac{d}{dx}\left(\int_0^x \sqrt{t + \sqrt{t}}\, dt \right) = \sqrt{x + \sqrt{x}}$

End of Example#2

Example #3 Let $h(x) = \int_0^{x^2} \sqrt{1 + r^3}\, dr$

Find $h'(x)$. That is, find $\frac{d}{dx} h(x)$

Solution Recognize that this is a nested function

$$\frac{d}{dx} h(x) = \frac{d}{dx}\, \text{outer}(\text{inner}(x))$$

chain rule

$$= \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

$$= \sqrt{1 + (x^2)^3} \cdot 2x$$

$$= \sqrt{1 + x^6} \cdot 2x$$

Chain rule details

$\text{inner}(x) = x^2$

$\text{inner}'(x) = 2x$

$\text{outer}(\ ) = \int_0^{(\ )} \sqrt{1 + r^3}\, dr$

$\text{outer}'(\ ) = \sqrt{1 + (\ )^3}$

End of Example #3

End of Meeting