MATH 2301 Section 110 (Barsamian) Meeting #52 (Mon Apr 24, 2023)

Today: Section 5.4 Fundamental theorem of Calculus
Section 5.5 The Substitution Rule

Tomorrow: Last Recitation

Tues May 2: Final Exam 4:40pm - 6:30pm, room to be announced

Good News: OU MATH 2301 web page (not mine)
has some sample old final exams

Bad News: Those exams have some topics that are
not part of our course
(for example: Newton's Method)
They were NOT written by the person who
will be writing this semesters exam.

My Suggestion for Study Strategy

1. Review (and resolve) old quiz + exam problems that you were successful on. You'll remember it quickest
2. Review (and resolve) old quiz + exam problems that you got wrong. Read comments from me.
3. Solve suggested problems.
   - easy one from each section
   - moderate one from each section
   - hard one from each section

   Repeat

Meeting Part I Fundamental Theorem of Calculus (Section 5.4)

Fundamental Theorem of Calculus

Part 1

$$\frac{d}{d\cancel{x}}\left(\int_a^x f(t)\,dt\right) = f(x)$$

$\frac{d}{dx}$

Part 2

the evaluation theorem

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left(\int f(x)\,dx\right)\Bigg|_a^b$$

where F is an antiderivative of f

Special version of this is called the Net Change Theorem

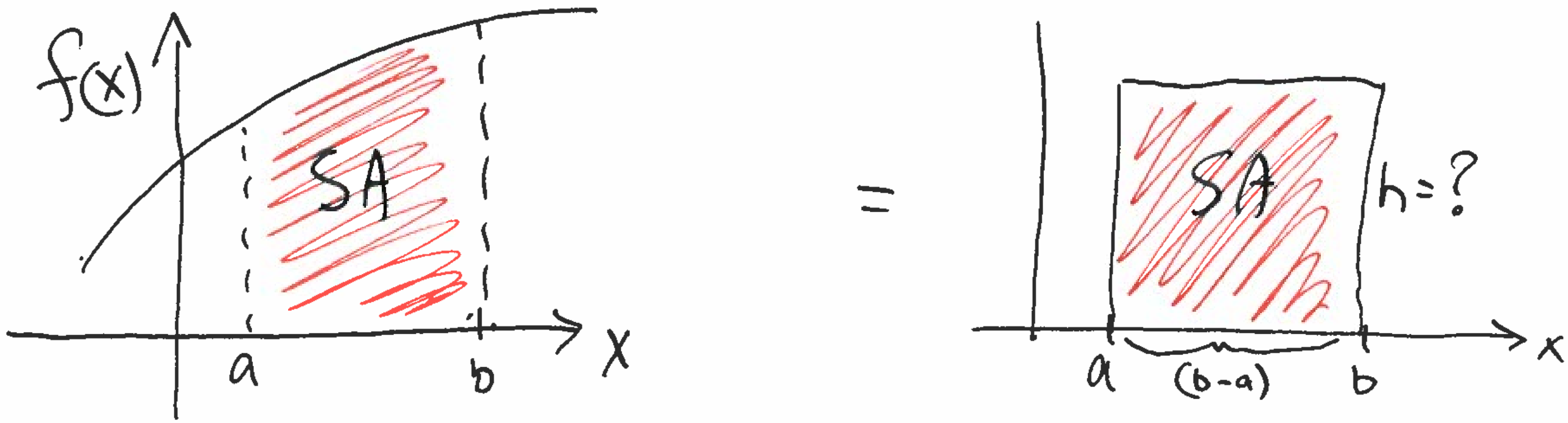
$$\Delta F = F(b) - F(a) = \int_a^b F'(x)\,dx$$

Leftover topic from Section 5.4

Geometric Question

Given a continuous function $f(x)$ on an interval $[a,b]$,

Consider the signed area between $f(x)$ and x axis from a to b



If we want a rectangle on the same interval $[a,b]$ to have the same area, how high would that rectangle have to be?

Rectangle area = width $\cdot$ h = $(b-a)h$

Signed area under graph = $\displaystyle\int_a^b f(x)\,dx$

Set these equal and solve for h

$$(b-a)h = \int_a^b f(x)\,dx$$

Divide by b-a

$$h = \frac{1}{(b-a)} \int_a^b f(x)\,dx$$

That answers our question.

This special h gets a fancy name.

The average value of f(x) on the interval [a,b]

Example

Find average value of $\sqrt[3]{x}$ on interval $[8, 27]$

Solution: rewrite this: $\sqrt[3]{x} = x^{1/3}$

$$h = \frac{1}{27-8} \int_8^{27} x^{1/3}\,dx$$

$$= \frac{1}{19} \left[\int x^{1/3}\,dx \right] \Big|_8^{27}$$

Indefinite Integral Details

$$\int x^{1/3}\,dx = \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} + C$$

$$= \frac{x^{4/3}}{4/3} + C$$

$$= \frac{3}{4} x^{4/3} + C$$

$$= \frac{1}{19} \left(\frac{3}{4} x^{4/3} + C \right) \Big|_8^{27}$$

$$= \frac{1}{19} \left[\left(\frac{3}{4} \cdot 27^{4/3} + \cancel{C} \right) - \left(\frac{3}{4} \cdot 8^{4/3} + \cancel{C} \right) \right]$$

$$= \frac{1}{19} \left[\left(\frac{3}{4} \cdot 81 \right) - \left(\frac{3}{4} \cdot 16 \right) \right]$$

$$27^{4/3} = 27^{\frac{1}{3}\cdot 4} = \left(27^{1/3}\right)^4 = 3^4 = 81$$

$$8^{4/3} = 8^{\frac{1}{3}\cdot 4} = \left(8^{1/3}\right)^4 = 2^4 = 16$$

$$= \frac{1}{19} \cdot \frac{3}{4}(81-16)$$

$$= \frac{1}{19} \cdot \frac{3}{4} \cdot 65$$

$$h = \frac{195}{76}$$

<u>Example</u> Find average value of $y = \frac{1}{x}$ on interval $[1, 4]$

<u>Solution</u>

$$h = \frac{1}{4-1} \int_1^4 \frac{1}{x}\,dx$$

$$= \frac{1}{3}\left(\int \frac{1}{x}\,dx\right)\Big|_1^4$$

$$= \frac{1}{3}\left(\ln|x| + c\right)\Big|_1^4$$

$$= \frac{1}{3}\left(\ln|4| \cancel{+c} - \ln|1| \cancel{+c}\right)$$

$$= \frac{1}{3}\left(\ln(4) - \cancel{\ln(1)}^{\,0}\right)$$

$$= \frac{\ln(4)}{3}$$

Meeting Part 2 The Substitution Rule (Section 5.5)

Used for finding indefinite integrals

$$\int f(x)\,dx$$

when the integrand involves a nested function (composition of functions)

Step 1 Identify the Inner Function call it u. Circle the equation $\text{inner}(x) = u$

Step 2 Build the expression $dx = \frac{1}{u'}du$ This will involve first finding u' Circle the equation $dx = \frac{1}{u'}du$

Step 3 Substitute, Cancel, Simplify
Substitute the circled items into the integral, cancel as much as possible, and simplify using constant multiple rule. Result of Step 3 should be a new basic integral involving just u, no x.

Step 4 Integrate

Step 5 Substitute back using $\text{inner}(x) = u$. Present answer clearly

Example #1 Find $\int x\cos x^2\,dx$

Step 1 Identify inner function

~~inner(x) = x^2~~ $x^2 = u$

Step 2 Build $dx = \frac{1}{u'}du$

$u = x^2$

$u' = 2x$

$dx = \frac{1}{2x}du$

Step 3 Substitute, Cancel, Simplify

$$\int x\cos(x^2)\,dx \underset{\text{Substitute}}{=} \int x\cos(u)\frac{1}{2x}du \underset{\text{Cancel}}{=} \int \frac{\cos(u)}{2}du \underset{\text{Simplify}}{=} \frac{1}{2}\int\cos(u)\,du$$

Step 4 Integrate

$$\frac{1}{2}\int\cos(u)\,du = \frac{1}{2}\left(\sin(u) + \underset{\text{Could be any real number}}{C}\right) = \frac{1}{2}\sin(u) + \underset{\text{could be any real number}}{\frac{1}{2}C} = \frac{1}{2}\sin(u) + \underset{\text{D can be any real number}}{D}$$

Step 5 Substitute back

$$\frac{1}{2}\sin(u) + D = \frac{1}{2}\sin(x^2) + D$$

Conclusion $\int x\cos(x^2)\,dx = \frac{1}{2}\sin(x^2) + D$

~~#5~~ **Example #2** Find $\int \frac{(\ln(x))^5}{x} dx$

Solution

Step 1 inner function $\ln(x) = u$

Step 2 build $dx = \frac{1}{u'} du$

$u = \ln(x)$

so $u' = \frac{1}{x}$

So $\frac{1}{u'} = \frac{1}{1/x} = x$

$dx = \frac{1}{u'} du$

$dx = x\,du$

Step 3 Substitute, Cancel, Simplify

$$\int \frac{(\ln(x))^5}{x} dx \underset{\text{Substitute}}{=} \int \frac{(u)^5}{x} x\,du \underset{\text{cancel}}{=} \int u^5 du$$

Step 4 Integrate

$$\int u^5 du = \frac{u^6}{6} + C$$

Step 5 Substitute back

$$\frac{u^6}{6} + C = \frac{(\ln(x))^6}{6} + C$$

Conclusion

$$\boxed{\int \frac{(\ln(x))^5}{x}\,dx = \frac{(\ln(x))^6}{6} + C}$$

End of Example

End of Meeting