MATH 2301 Section 110 (Barsamian) Meeting #55, Fri Apr 28, 2023

Today Finish Section 5.5 The Substitution Rule

Discuss The Normal Probability **Density** and **Distribution**

Tues May 2, Cumulative Final Exam, 4:40-6:40pm, **here**.

Alternate Seats **and** Alternate Rows

Meeting Part 1 Definite Integrals that Involve Substitution (Section 5.5)

Example #1 5.5 #37

Find the definite integral $\int_0^1 \cos\left(\frac{\pi t}{2}\right)dt$

Solution

$$\int_{t=0}^{t=1} \cos\left(\frac{\pi t}{2}\right)dt \underset{\text{FTC}}{=} \left(\int \cos\left(\frac{\pi t}{2}\right)dt\right)\Bigg|_{t=0}^{t=1}$$

Indefinite Integral Details

$$\int \cos\left(\frac{\pi t}{2}\right)dt \underset{\text{Substitute}}{=} \int \cos(u)\frac{2}{\pi}du$$

$$\underset{\text{Simplify}}{=} \frac{2}{\pi}\int \cos(u)du$$

$$\underset{\text{Integrate}}{=} \frac{2}{\pi}\left(\sin(u)+C\right)$$

$$= \frac{2}{\pi}\sin(u) + K$$

$$\underset{\text{Substitute}}{=} \frac{2}{\pi}\sin\left(\frac{\pi t}{2}\right) + K$$

Substitution details

$\frac{\pi t}{2} = u$

$u' = \frac{d}{dt}\left(\frac{\pi t}{2}\right) = \frac{\pi}{2}$

$dt = \frac{1}{u'}du$

$dt = \frac{1}{\pi/2}du$

$dt = (2/\pi)du$

$$= \left(\frac{2}{\pi}\sin\left(\frac{\pi t}{2}\right) + K\right)\Bigg|_{t=0}^{t=1}$$

$$= \left(\frac{2}{\pi}\sin\left(\frac{\pi(1)}{2}\right)\cancel{+K}\right) - \left(\frac{2}{\pi}\sin\left(\frac{\pi(0)}{2}\right)\cancel{+K}\right)$$

$$= \frac{2}{\pi}\underbrace{\sin\left(\frac{\pi}{2}\right)}_{1} - \frac{2}{\pi}\underbrace{\sin(0)}_{0}$$

$$= \frac{2}{\pi}$$

End of Example

Alternate Solution (Book's Method)

$$\int_{t=0}^{t=1} \cos\left(\frac{\pi t}{2}\right) dt = \int_{u=0}^{u=\pi/2} \cos(u)\, \frac{2}{\pi} du$$

$$\underset{\text{Simplify}}{=} \frac{2}{\pi} \int_{u=0}^{u=\pi/2} \cos(u)\, du$$

$$= \frac{2}{\pi} \left(\int \cos(u)\, du \right) \Bigg|_{u=0}^{u=\pi/2}$$

$$= \frac{2}{\pi} \left[\left(\sin(u) + C \right) \Big|_{u=0}^{u=\pi/2} \right]$$

$$= \frac{2}{\pi} \left[\left(\underbrace{\sin(\pi/2)}_{1} + \cancel{C} \right) - \left(\underbrace{\sin(0)}_{0} + \cancel{C} \right) \right]$$

$$= \frac{2}{\pi}$$

End of Example

Substitution Involving both the integrand and the endpoints

Substitution involving the integrand

$$\frac{\pi t}{2} = u$$

$$u' = \frac{d}{dt}\left(\frac{\pi t}{2}\right) = \frac{\pi}{2}$$

$$dt = \frac{1}{u'} du = \frac{1}{\pi/2} du = \frac{2}{\pi} du$$

Convert the endpoints using the equation $u = \frac{\pi t}{2}$

when $t = 0$, $u = \frac{\pi(0)}{2} = 0$

when $t = 1$ $u = \frac{\pi(1)}{2} = \frac{\pi}{2}$

Another Example 5.5#39 Find $\int_0^1 \sqrt[3]{1+7x}\,dx$

(4)

Solution (Mark's method)

$$\int_{x=0}^{x=1} (1+7x)^{1/3}\,dx \underset{\text{FTC}}{=} \left(\int (1+7x)^{1/3}\,dx\right)\Bigg|_{x=0}^{x=1}$$

Indefinite Integral details

$$\int (1+7x)^{1/3}\,dx = \int (u)^{1/3}\,\frac{1}{7}\,du = \frac{1}{7}\int u^{1/3}\,du$$

$1+7x=u$

$u' = \frac{d}{dx}(1+7x) = 7$

$dx = \frac{1}{u'}\,du$

$dx = \frac{1}{7}\,du$

Substitution details

$$= \frac{1}{7}\left(\frac{u^{1/3+1}}{\frac{1}{3}+1} + C\right) = \frac{1}{7}\,\frac{u^{\frac{4}{3}}}{\frac{4}{3}} + \frac{1}{7}\cdot C$$

$$= \frac{1}{7}\cdot\frac{3}{4}\cdot u^{4/3} + K$$

$$= \frac{3}{28}\,u^{4/3} + K$$

$$= \frac{3}{28}(1+7x)^{4/3} + K$$

$$= \left(\frac{3}{28}(1+7x)^{4/3} + K\right)\Bigg|_{x=0}^{x=1}$$

$$= \left(\frac{3}{28}(1+7(1))^{4/3} + \cancel{K}\right) - \left(\frac{3}{28}(1+7(0))^{4/3} + \cancel{K}\right)$$

$$= \frac{3}{28}(8)^{4/3} - \frac{3}{28}(1)^{4/3}$$

$$= \frac{3}{28}\left((8)^{4/3} - (1)^{4/3}\right)$$

$8^{\frac{4}{3}} = \left(8^{\frac{1}{3}}\right)^4 = (2)^4 = 16$

$$= \frac{3}{28}(16 - 1)$$

$$= \frac{3}{28}\cdot 15 = \frac{45}{28}$$

End of Example

Same example, using Buck's Solution Method

$$\int_{x=0}^{x=1} (1+7x)^{1/3}\,dx = \int_{u=1}^{u=8} u^{1/3}\frac{1}{7}\,du = \frac{1}{7}\int_{u=1}^{u=8} u^{1/3}\,du =$$

Substitution details

$1+7x = u$

$dx = \frac{1}{7}du$

Convert the endpoints

when $x=0$, $u = 1+7(0) = 1$

$x=1$, $u = 1+7(1) = 8$

$$= \frac{1}{7}\left(\int u^{1/3}du\right)\Big|_{u=1}^{u=8} = \frac{1}{7}\left[\left(\frac{u^{4/3}}{4/3}+C\right)\Big|_{u=1}^{u=8}\right] =$$

$$= \frac{1}{7}\cdot\left[\left(\frac{3}{4}u^{4/3}+C\right)\Big|_{u=1}^{u=8}\right] = \frac{1}{7}\left[\left(\frac{3}{4}(8)^{4/3}+\cancel{C}\right)-\left(\left(\frac{3}{4}\right)(1)^{4/3}+\cancel{C}\right)\right]$$

$$= \frac{1}{7}\cdot\frac{3}{4}\cdot\left(8^{4/3}-1^{4/3}\right) = \frac{3}{28}(16-1) = \frac{3}{28}\cdot 15 = \frac{45}{28}$$

End of Example

Meeting Part 2 Normal Probability Density & Distribution

On Wednesday: the function $f(x) = e^{(x^2)}$ has an antiderivative but it cannot be written as a finite combination of basic functions.

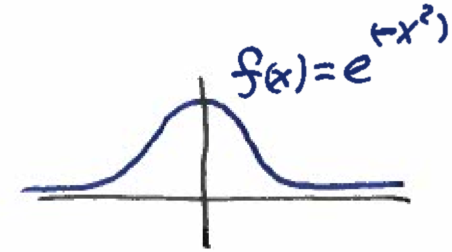
~~An ant~~ A particular antiderivative is

$$F(x) = \int_{t=0}^{t=x} e^{(t^2)}\,dt$$

By F.T.C. fundamental theorem

$$F'(x) = \frac{d}{dx}\left(\int_{t=0}^{t=x} e^{(t^2)}\,dt\right) = e^{x^2} = f(x)$$

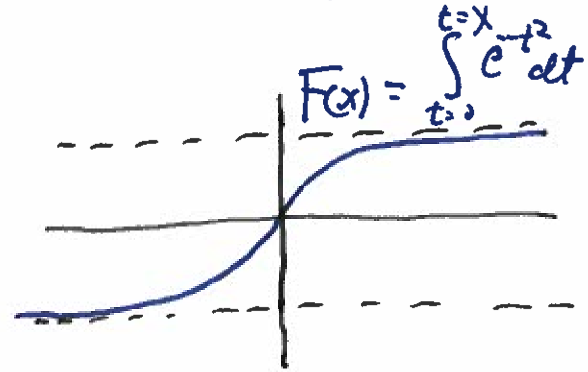
Another function with similar property $f(x) = e^{(-x^2)}$



Bell-shaped Curve

a particular antiderivative is

$$F(x) = \int_{t=0}^{t=X} e^{(-t^2)}\,dt$$



End of Discussion of Normal Probability

End of Meeting

End of Semester!!