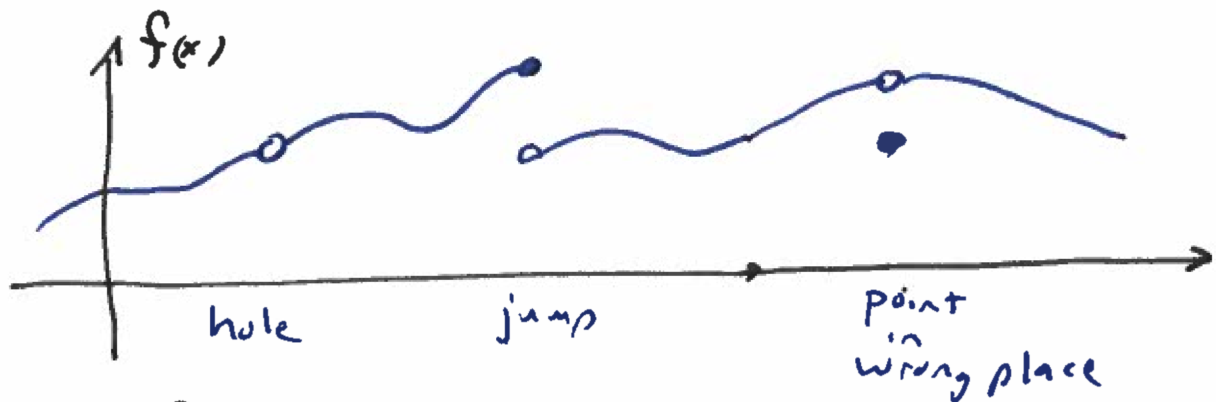
MATH 2301 Day 6 (Wed Jan 25 Lecture)

Quiz Q1 Friday over 1.3, 1.4. Prepare by <u>writing</u> solutions to the suggested HW problems

<u>Section 1.5</u> Continuity

It is easy to spot weird behavior in a graph



Suppose f(x) is described by a <u>formula</u>, not by a graph.

It would be nice to have some math tools that would articulate analogous behavior in a function defined by a <u>formula</u>.

That is the idea of <u>Continuity</u>.

Definition

words: f is continuous at a

usage: f is a function, and a is a real number

meaning: $\lim_{x \to a} f(x) = f(a)$

Notice that for this equation to be true a bunch of stuff must be true

(1) $\lim_{x \to a} f(x)$ must exist

(1a) $\lim_{x \to a^-} f(x)$ must exist

(1b) $\lim_{x \to a^+} f(x)$ must exist

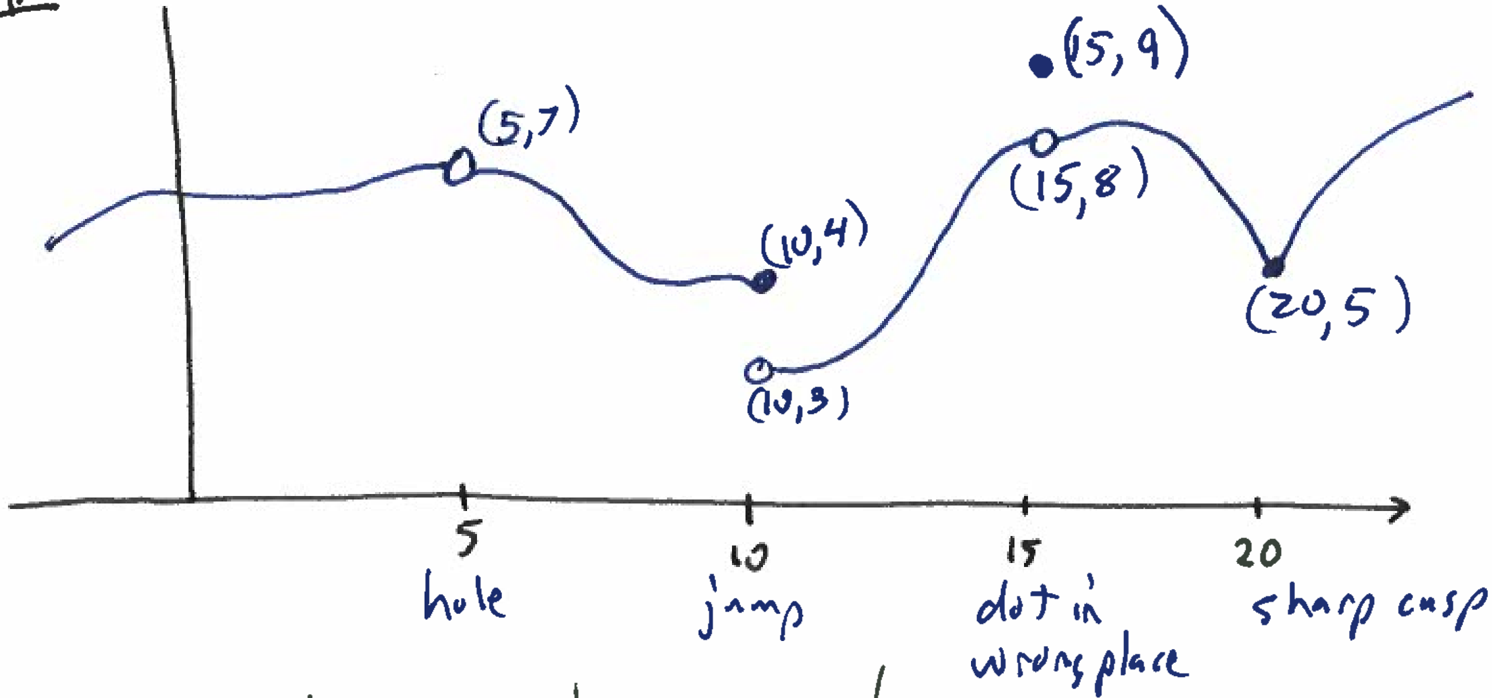
(1c) the left & right limits must match

(2) $f(a)$ must exist

(3) The limit must equal the y value

$$\lim_{x \to a} f(x) = f(a)$$

Example



	$x=5$	$x=10$	$x=15$	$x=20$
(1) $\lim_{x\to a} f(x)$ exists	yes $L=7$	no	yes $L=8$	yes $L=5$
(1a) left limit exists (1b) right limit exists (1c) left limit = right limit	yes 7 yes 7 yes 7	yes $L=4$ yes $L=3$ no	8 8 8	5 5 5
(2) $f(a)$ exists	no	yes $y=4$	yes $y=9$	yes $y=5$
(3) limit equals y value	no	no	no $L\neq y$	yes $L=y$

One Sided Continuity

<u>Definition</u>

words: f is continuous from the left at a

meaning: $\lim_{x\to a^-} f(x) = f(a)$

Definition words: f is continuous from the right at a

meaning: $\lim_{x\to a^+} f(x) = f(a)$

Definition

<u>words</u> f is continuous on interval I

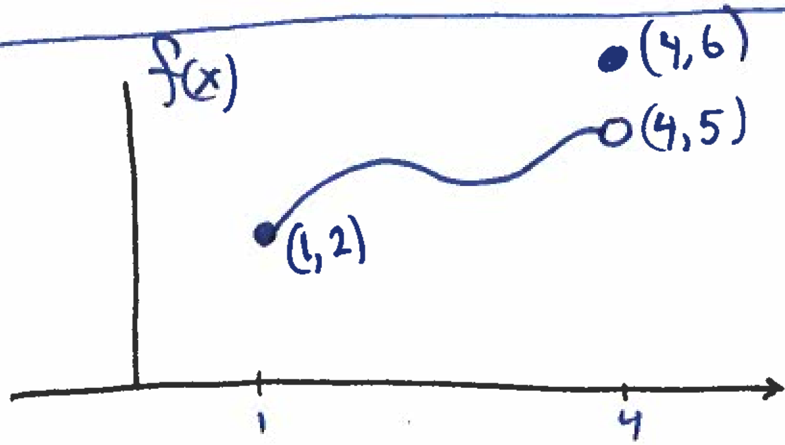
↑ could be (a,b) or $[a,b]$ or $[a,b)$ or $(a,b]$

open interval — closed interval — half open intervals

<u>meaning</u> f is continuous at all $x=c$ where $c \in I$

$\in$: is an element of

<u>Example:</u>



f is continuous on $(1,4)$ true

f is continuous on $[1,4)$ most descriptive

f is <u>not</u> continuous on $[1,5]$

Section 1.5 boxed item [5] Theorem

(a) a polynomial is continuous everywhere

(b) a rational function is continuous everywhere that it is defined. That is, it is continuous on its domain.

In other words. A rational function is continuous everywhere <u>except</u> at x values that cause denominator $= 0$

This is a result of the Chapter 1.4 ~~Derivative Rule~~ Limit Rule

<u>Direct substitution property</u> (from 1.4)

If f is polynomial or rational function and $x = a$ is in its domain

the $\lim_{x \to a} f(x) = f(a)$

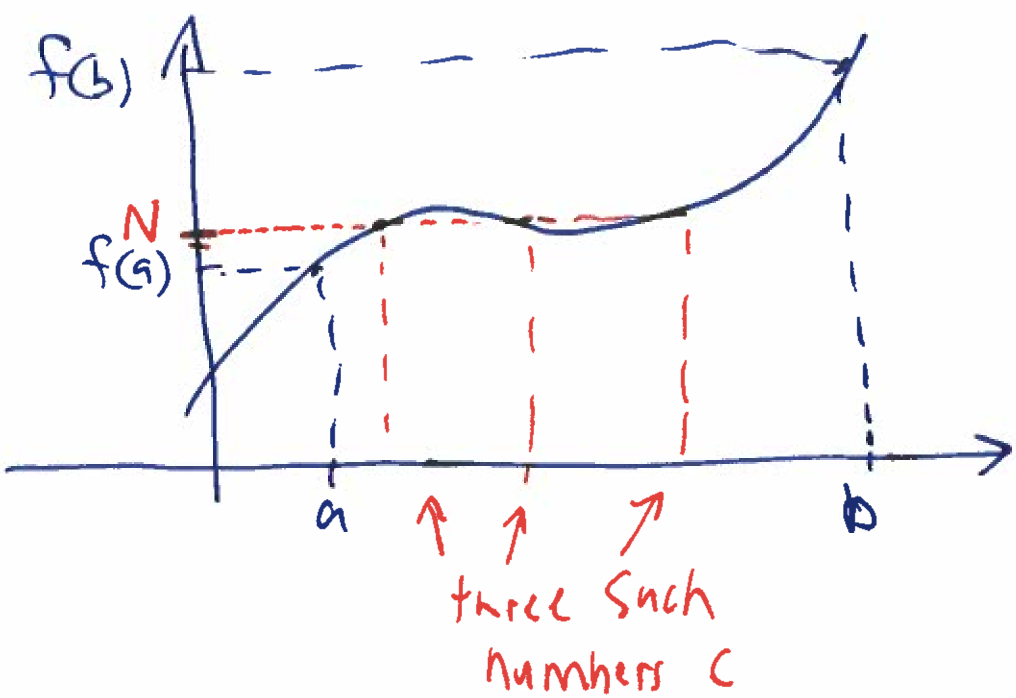
The Intermediate Value Theorem

Section 1.5 Boxed Item [9] The Intermediate value theorem (rewritten)

If f is continuous on the closed interval $[a, b]$

and $f(a) < N < f(b)$

Then there exists (at least one) number $c \in (a, b)$ such that $f(c) = N$



Example

Show that there is a root of the equation

$$x^7 - 23x^2 + 15x - 11 = 0$$

(Show that there is an x that makes this true)

Solution

Let $f(x) = x^7 - 23x^2 + 15x - 11 = 0$

Observe: f is continuous everywhere

Let $a = 0$ then $f(a) = 0^7 - 23(0)^2 + 15(0) - 11 = -11$

Let $b = 10$ then $f(b) = 10^7 - 23(10)^2 + 15(10) - 11 =$ some positive number

Observe

$$-11 < 0 < \text{some positive number}$$

$$f(a) < N < f(b).$$

So by intermediate value theorem there exists some $x = c$ such that $f(c) = N = 0$