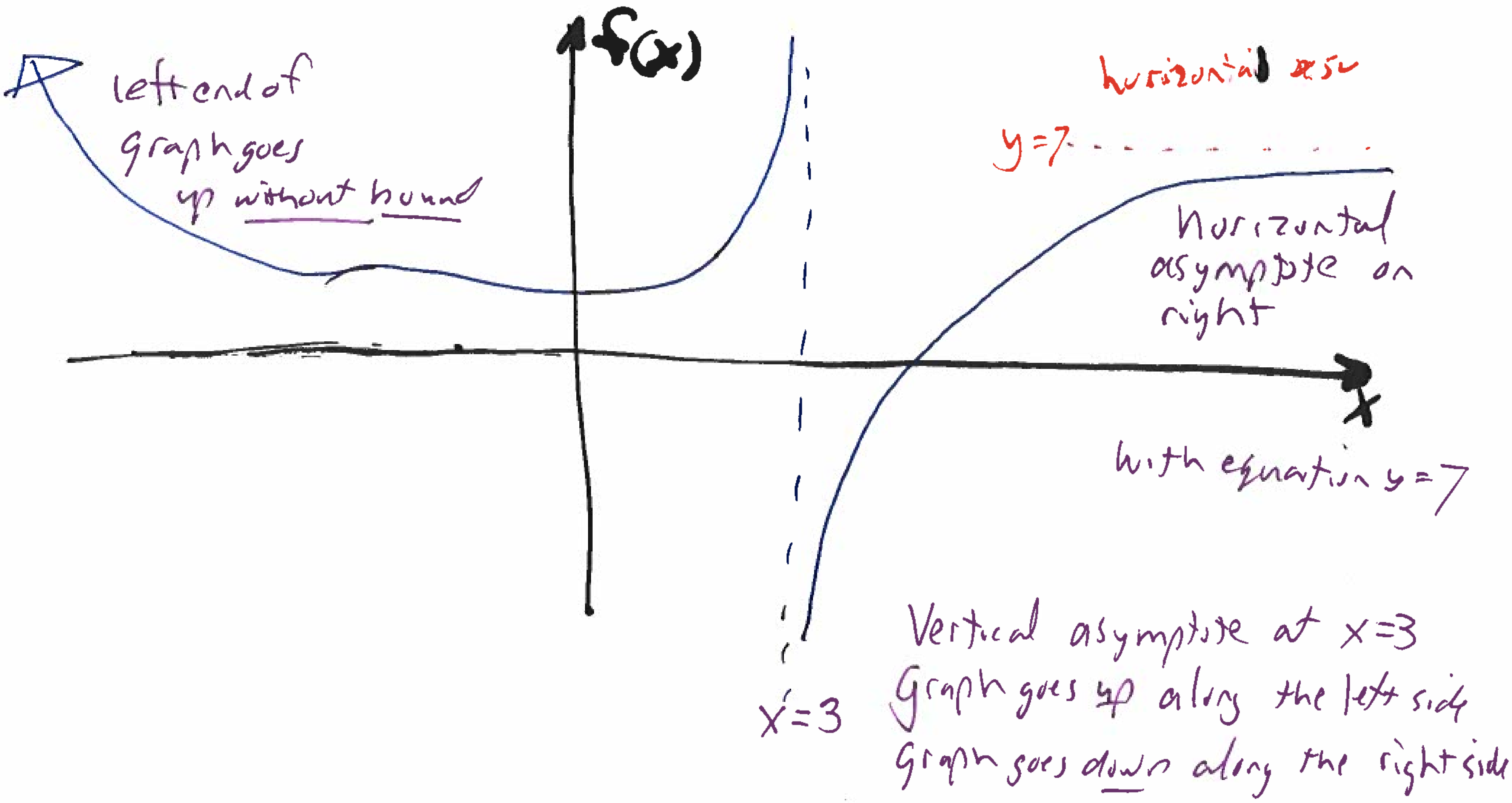
MATH 2301 Day 7 (Fri Jan 27)

Section 1.6 Limits Involving Infinity

Asymptotic Behaviour is easy to spot in a graph



We would like to have corresponding terminology for functions given by ~~[illegible]~~ formulas, not graphs.

That's the idea of Infinite Limits and Limits at infinity.

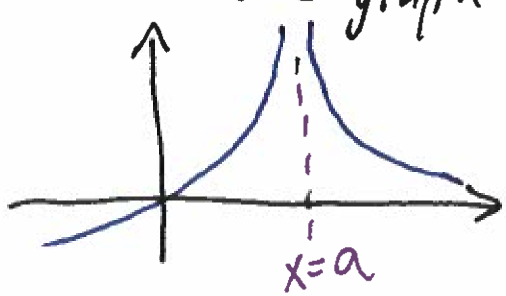
Definition of infinite limit

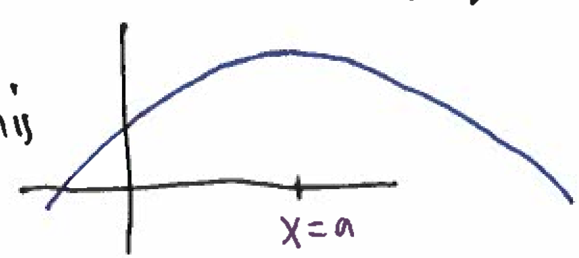
Symbol: $\lim_{x \to a} f(x) = \infty$

usage: a is a number, $f(x)$ is a function.

meaning (informal definition) When x gets closer & closer to a
but not equal to a,
the y values $f(x)$ get more & more positive
without bound

graphical significance: The graph of f has a vertical asymptote with line equation $x=a$, and the graph goes up on both sides of that asymptote



(but not this 

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There are natural variations

$$\lim_{x \to a} f(x) = -\infty$$

or

$$\lim_{x \to a^-} f(x) = \infty$$

or

$$\lim_{x \to a^+} f(x) = -\infty$$

Example

let $f(x) = \cancel{\frac{x+5}{x+4}} \quad \frac{x+5}{x+2}$

use a table of x, y values to guess $\lim_{x \to -2^-} f(x)$

Solution

x	$y = \frac{x+5}{x+2}$
-2.1	$f(-2.1) = \frac{(-2.1)+5}{(-2.1)+2} = \frac{2.9}{-0.1} = \frac{29}{-1} = -29$
-2.01	$f(-2.01) = \frac{(-2.01)+5}{(-2.01)+2} = \frac{2.99}{-0.01} = \frac{299}{-1} = -299$
-2.001	$f(-2.001) = \frac{(-2.001)+5}{(-2.001)+2} = \frac{2.999}{-0.001} = \frac{2999}{-1} = -2999$

$x \to -2^-$

$y \to -\infty$

$$\lim_{x \to -2^-} \frac{x+5}{x+2} = -\infty$$

Same problem find $\lim_{x \to -2^-} \frac{x+5}{x+2}$

incredibly cool by scrutinizing the relative sizes of the numerator, denominator, and the ratio.

$$\lim_{x \to -2^-} \frac{x+5}{x+2} = -\infty$$

numerator will be close to 3

denominator will be negative number very close to 0.

this tells us that x is close to -2 but $x < -2$

So the ratio will be a huge, negative number