MATH 2301 (Barsamian) Day 8 (Mon Jan 30 Lecture)

Recitation Tomorrow (Tue Jan 31) Solving problems (like last week's Recitation) See course web page for your assigned problem.

(**Prepare In Advance!**)

Common Mistakes on Quiz Q1

- not explaining the cancellation step (The most important concept of the first month of the course!)
- not showing clearly the result of the cancellation
- omitting limit symbol (or including it where it does not belong)

- Combining cancelling steps that cannot be combined

Example

Fiona

$$\cancel{\frac{49+\cancel{3}-49}{\cancel{3}} = \frac{49-49}{1} = \frac{0}{1}}$$

$$= \frac{49+1-49}{1} = \frac{1}{1} = 1$$

$$\frac{10+11+12}{11} = \frac{10+\cancel{11}+12}{\cancel{11}} = \frac{10+1+12}{1} = \frac{23}{1} = 23$$

Bernhard

$$\frac{10+11+12}{11} = \frac{33}{11} = 3$$

Example

Frick: $\lim_{h\to 0} \frac{\cancel{49} + h - \cancel{49}}{h} = \lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}} = \lim_{h\to 0} 1 = 1$

can cancel because $h \neq 0$

Frack: $\lim_{h\to 0} \frac{49 + \cancel{h} - 49}{\cancel{h}} = \lim_{h\to 0} \frac{49+1-49}{1} = \lim_{h\to 0} \frac{1}{1} = 1$

can cancel because $h \neq 0$

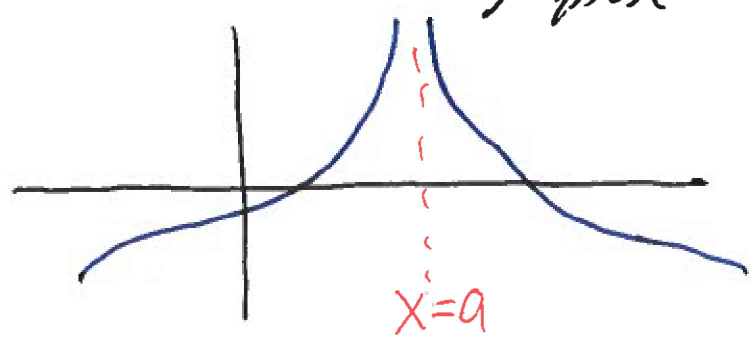
WackyJack: $\lim_{h\to 0} \frac{\cancel{49+h-49}}{\cancel{h}} = \lim_{h\to 0} 1 = 1$

Continuing Section 1.6 Limits involving Infinity

Friday We discussed Infinite Limits at $x = a$ where a is a real number constant.

$$\lim_{x \to a} f(x) = \infty \quad \longleftrightarrow$$

graph of $f(x)$ has a vertical asymptote with line equation $x = a$ and graph goes up along both sides of the asymptote

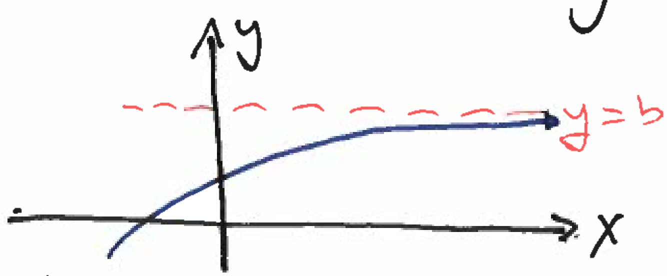


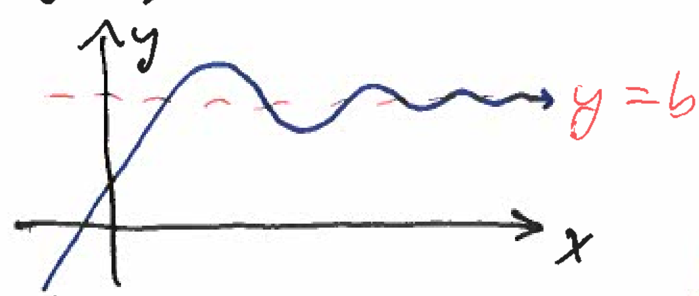
Today "Limits at Infinity"

That means $\lim_{x\to\infty} f(x)$ or $\lim_{x\to-\infty} f(x)$

Informal Definition of Limit at Infinity

- Symbol: $\lim_{x\to\infty} f(x) = b$
- spoken: The limit, as x goes to infinity, of $f(x)$ is b.
- meaning (informal): when x gets more & more positive without bound, the values of $f(x)$ get closer & closer to b and may actually equal b
- pictures





- graphical significance: graph has horiz. asymptote on right with line equation $y=b$.

Example

find $\lim_{x\to\infty} \dfrac{7x^2-7x-42}{3x^2-9x-30}$

that is $f(x) = \dfrac{7x^2-7x-42}{3x^2-9x-30}$

find $\lim_{x\to\infty} f(x)$

Method #1 Identify dominant terms

really huge

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{7x^2-7x-42}{3x^2-9x-30} = \lim_{x\to\infty} \frac{7\cancel{x^2}}{3\cancel{x^2}} = \lim_{x\to\infty} \frac{7}{3} = \frac{7}{3}$$

x is getting huge & positive

really huge

keep the dominant terms

Since $x\to\infty$ we know $x \neq 0$ so we can cancel

this tells us that graph of $f(x)$ has horiz asympt on right with line equation $y=\frac{7}{3}$

Method #2 Book's Method   know how to do this

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{7x^2-7x-42}{3x^2-9x-30}$$

divide top & bottom by highest power of x that appears anywhere

$$= \lim_{x\to\infty} \frac{\frac{7x^2}{x^2} - \frac{7x}{x^2} - \frac{42}{x^2}}{\frac{3x^2}{x^2} - \frac{9x}{x^2} - \frac{30}{x^2}}$$

can cancel because $x \neq 0$

$$= \lim_{x\to\infty} \frac{7 - \frac{7}{x} - \frac{42}{\cancel{3}x^2}}{3 - \frac{9}{x} - \frac{30}{x^2}}$$

$$= \frac{\lim_{x\to\infty} 7 - \lim_{x\to\infty} \frac{7}{x} - \lim_{x\to\infty} \frac{42}{x^2}}{\lim_{x\to\infty} 3 - \lim_{x\to\infty} \frac{9}{x} - \lim_{x\to\infty} \frac{30}{x^2}}$$

$$= \frac{7 - 0 - 0}{3 - 0 - 0}$$

$$= \frac{7}{3}$$

Same result

Example

this is $\infty - \infty$ indeterminate form

$$\lim_{x\to\infty}\left(\sqrt{25x^2+2x} - 5x\right) =$$

$$= \lim_{x\to\infty}\left(\sqrt{25x^2+2x} - 5x\right)\frac{\left(\sqrt{25x^2+2x} + 5x\right)}{\left(\sqrt{25x^2+2x} + 5x\right)}$$

$$= \lim_{x\to\infty}\frac{\sqrt{25x^2+2x}\sqrt{25x^2+2x} \cancel{-5x\sqrt{\quad}} \cancel{+5x\sqrt{\quad}} - 5x(5x)}{\sqrt{25x^2+2x} + 5x}$$

$$= \lim_{x\to\infty}\frac{\cancel{25x^2} + 2x - \cancel{25x^2}}{\sqrt{25x^2+2x} + 5x}$$

$$= \lim_{x\to\infty}\frac{2x}{\sqrt{25x^2+2x} + 5x}$$

finish in recitation