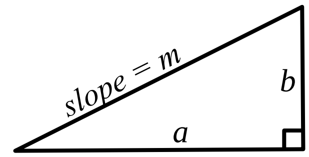
## Class Drill Part 1: The Idea Behind Newton's Method

**(a)** In the triangle shown, find an equation for the slope $m$ of the hypotenuse in terms of the lengths $a$ and $b$.




$m =$

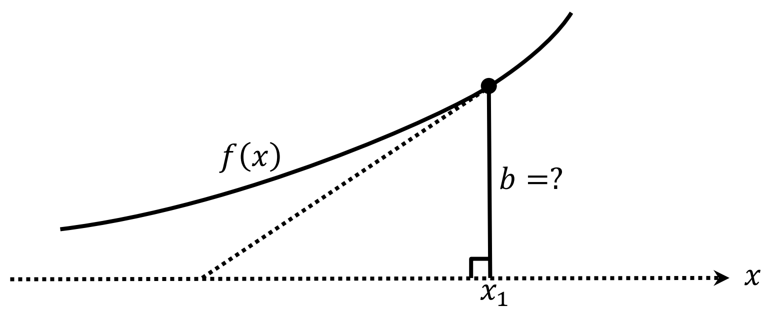
**(b)** Solve the equation for $a$ in terms of $m$ and $b$:

$a =$

**(c)** In the triangle shown, the upper right vertex lies on the graph of $f$.

How tall is the right leg?

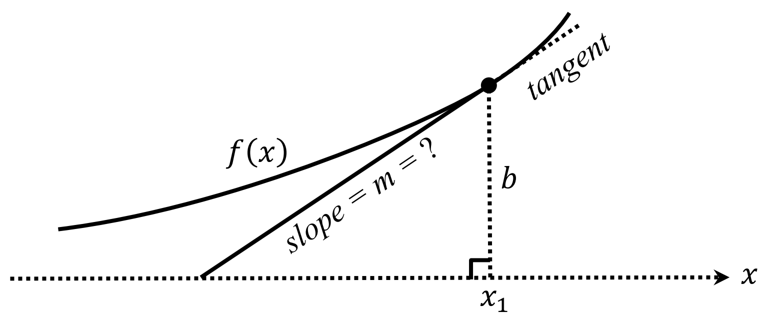
$b =$




**(d)** Suppose that the hypotenuse of the triangle is known to lie on the line that's tangent to the graph of $f$ at the point where $x = x_1$

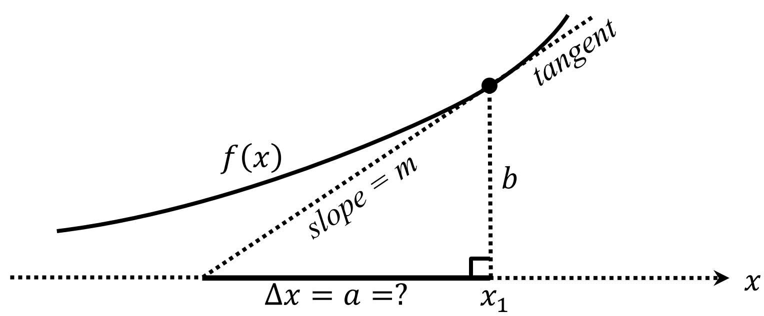
What is the hypotenuse slope $m$?

$m =$



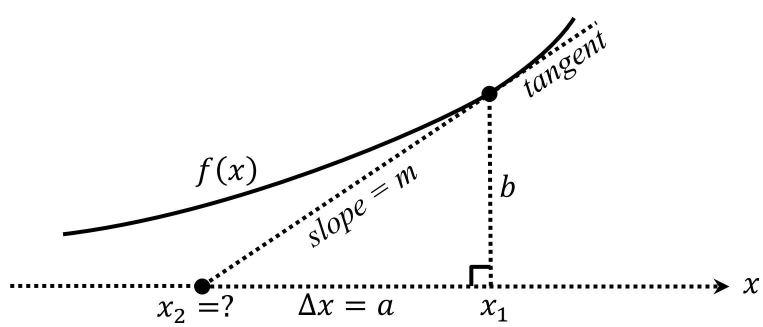

**(e)** For the same triangle, what is the base $\Delta x$?

$\Delta x = a =$




**(f)** For the same triangle, what is the $x$ coordinate $x_2$?

$x_2 =$




**The Class Drill continues on the next page ➔**

<u>**Class Drill Part 2: Using Newton's Method**</u>

**Newton's Method**

**Given:** A function $f$ that is differentiable on an interval $I$ and that has a root in $I$. That is, it is known that there exists a number $r$ somewhere in $I$ such that $f(r) = 0$.

**Goal:** Find an approximate value for the root $r$, accurate to $d$ decimal places.

**Step 1:** Choose a value $x_1$ as an initial approximation of the root. (This is often done by looking at a graph.)

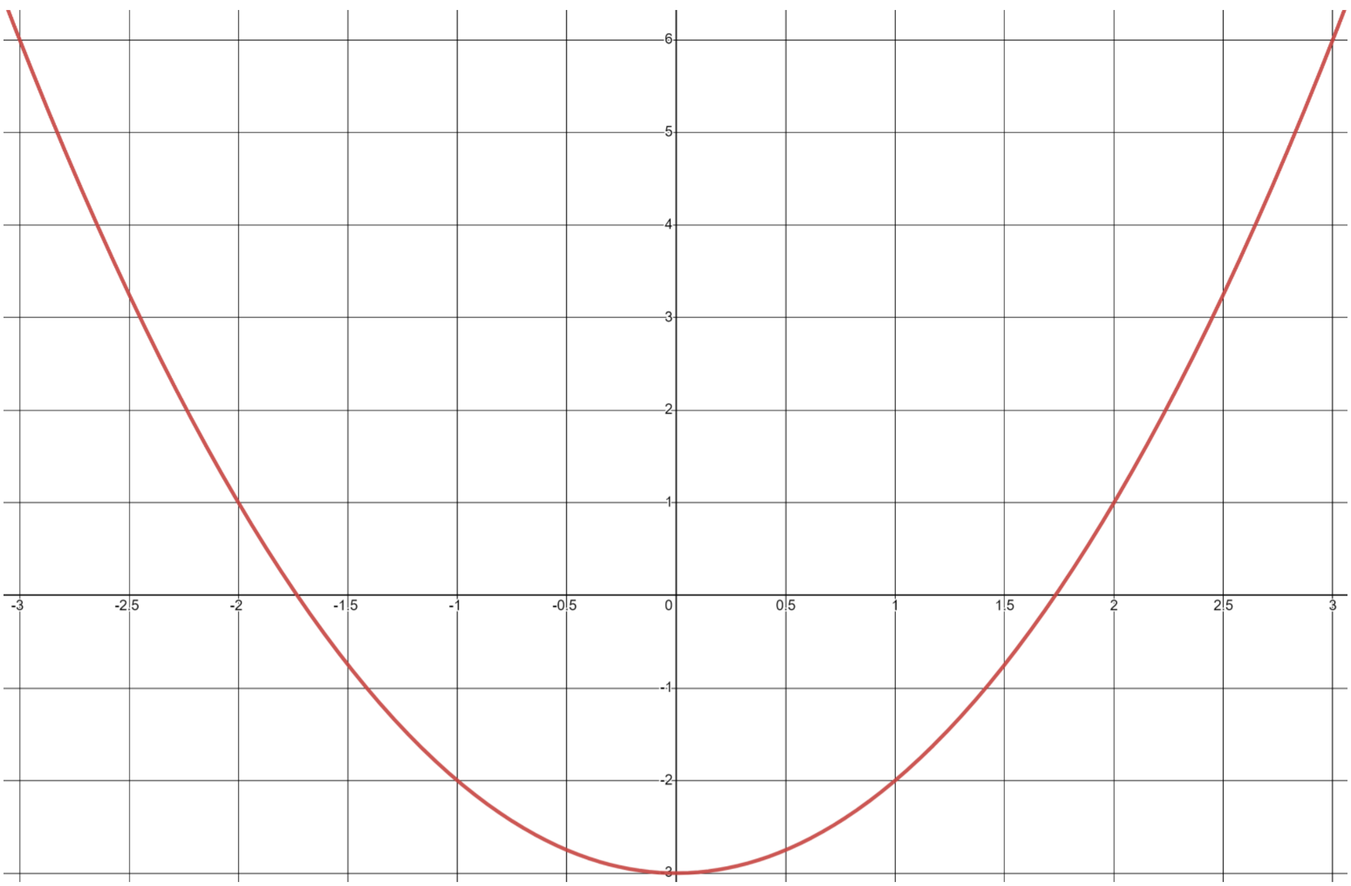
**Step 2:** Create successive approximations iteratively, as follows:

Given an approximation $x_n$, compute the next approximation $x_{n+1}$ by using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

**Step 3:** Stop the iterations when successive approximations do not differ in the first $d$ places after the decimal point. The last $x$ value computed is the approximation of $r$.

Let $f(x) = x^2 - 3$. Observe that the graph of $f(x)$ shows an $x$ intercept somewhere between $x = 1$ and $x = 2$. Using the terminology of roots, we would say that there is a root of $f$, that is, a number $r$ such that $f(r) = 0$, and that $r$ is somewhere between 1 and 2.




The goal is to use Newton's method to find an approximation for the root $r$. You will do the first three iterations only, using the initial approximation $x_1 = 3$. That is, you will find $x_2$, $x_3$, $x_4$.

For the function $f(x) = x^2 - 3$,

(a) Compute $f'(x)$

(b) Fill out the following table. (Do the details below.)

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ |
|---|---|---|---|---|
| 1 | $x_1 = 3$ | | | $x_2 =$ |
| 2 | $x_2 =$ | | | $x_3 =$ |
| 3 | $x_3 =$ | | | $x_4 =$ |
| 4 | $x_4 =$ | | | |

**The Class Drill continues on the next page ➔**

(C) A zoomed-in graph of $f(x)$ is shown below. You'll illustrate some of your results on this graph.

- Put a point at $(x_1, 0)$
- Put a point at $(x_1, f(x_1))$
- Draw the segment that connects $(x_1, 0)$ and $(x_1, f(x_1))$. This segment should be vertical.
- Put a point at $(x_2, 0)$.
- Draw the segment that passes through $(x_1, f(x_1))$ and $(x_2, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_1, f(x_1))$.
- Put a point at $(x_2, f(x_2))$
- Draw the segment that connects $(x_2, 0)$ and $(x_2, f(x_2))$. This segment should be vertical.
- Put a point at $(x_3, 0)$.
- Draw the segment that passes through $(x_2, f(x_2))$ and $(x_3, 0)$. This segment should appear to be tangent to the graph of $f(x)$ at the point $(x_2, f(x_2))$.