Linearizations and the Method for Finding a Linear Approximation
MATH 2301 (Barsamian)

Definition of the Linearization

Words: The ***linearization*** of $f(x)$

Meaning: The function $L(x)$ defined by the equation

$$L(x) = f(a) + f'(a)(x - a)$$

Graphical Significance: $L(x)$ describes the line that is tangent to the graph of $f(x)$ at $x = a$.

Method for Finding a Linear Approximation

Given: a function $f(x)$ and a hard x value called $\hat{x}$. (That is, it is not easy, or maybe even not possible, to compute $f(\hat{x})$ exactly by hand.)

Goal: Find an ***approximation*** for $f(\hat{x})$.

Steps:

- Identify the function $f(x)$
- Identify the hard x value, called $\hat{x}$.
- Identify an easy nearby x value, called a. That is, such that $f(a)$ is easy to compute.
- Build the ***linearization*** of $f(x)$ at a. That is, build the function
$$L(x) = f(a) + f'(a)(x - a)$$
- Use the ***linearization*** to compute the number $L(\hat{x})$. That is, compute
$$L(\hat{x}) = f(a) + f'(a)(\hat{x} - a)$$
(This should be an easy calculation.) This number $L(\hat{x})$ is the desired ***approximation*** for $f(\hat{x})$. It is called the ***linear approximation*** for $f(\hat{x})$