Rates of Change and Secant and Tangent Lines (Concepts from Section 2.1)

Definition of Average Rate of Change

Words: Average Rate of Change of *f* from *a* to *b*

Usage: a, b are real numbers, a < b, and f is a function that is continuous on the interval [a, b].

Meaning: the number $m = \frac{f(b) - f(a)}{b - 1}$

Graphical Significance: the number m is the slope of secant line that passes through points (a, f(a)) and (b, f(b))

Additional terminology: When the variable is t, representing time and the function f(t) is a position function, representing the position of an object at time t, then the average rate of change is called the *average velocity* from time a to time b.

Alternate presentation of average rate of change:

Words: Average Rate of Change of f from a to a + h

Usage: a, h are real numbers, $h \neq 0$, and f is a function that is continuous on an interval near a

Meaning: the number $m = \frac{f(a+h)-f(a)}{h}$

Graphical Significance: the number m is the slope of secant line that passes through points (a, f(a)) and (a + h, f(a + h))

Definition of Instantaneous of Change

Words: Instantaneous Rate of Change of *f* at *a*

Symbol: f'(a)

Spoken: The derivative of f at a

Usage: a is a real number and f is a function that is continuous near x = a

Meaning: the number $m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$

Additional terminology: When the variable is t, representing time and the function f(t) is a position function, representing the position of an object at time t, then the Instantaneous rate of change is called the *instantaneous velocity* at time a

Definition of line tangent to graph of f at x = a

The line that has these two properties

- contains the point(a, f(a)) (This point is called the point of tangency.)
- has slope m = f'(a) (This number is called the slope of the tangent line at x = a, but it is also called the slope of the graph of f(x) at x = a.)

General Point Slope Form of the Equation of the Tangent Line

The line tangent to the graph of f(x) at x = a has equation

$$(y - f(a)) = f'(a)(x - a)$$