MATH 2301 (Barsamian) Lecture #3 (Day #4) Fri Sep 1, 2023

Sit in front half of room

Sign In

Work on Homework

- Type answers into WebAssign
- But write solutions on paper, that you keep in a notebook

I recommend buying optional print book at College Bookstore

Recitation Tue Sep 5

Look for Recitation Assignments over the weekend

Extra Help

Kenny's Office Hours (Section 110)

Isaac's Office Hours (Section 100)

Academic Achievement Center

Continuing Section 1.4 Calculating Limits

[Example #1] consider $f(x) = \dfrac{x^2-4x}{x^2-3x-4} = \dfrac{x(x-4)}{(x+1)(x-4)}$

observe: $\lim\limits_{x\to 4}$ numerator $= \underbrace{\lim\limits_{x\to 4} x^2 - 4x}_{\text{limit of a polynomial}} = (4)^2 - 4(4) = 0$

direct sub property

$\lim\limits_{x\to 4}$ denominator $= \underbrace{\lim\limits_{x\to 4} (x+1)(x-4)}_{\text{limit of polynomial}} = (4+1)(4-4) = 5 \cdot 0 = 0$

Direct Sub property

So $\lim\limits_{x\to 4} \dfrac{x^2-4x}{x^2-3x+4}$ is in $\dfrac{0}{0}$ indeterminate form.

That is, basically, if we sub in $x=4$, we would get $\frac{0}{0}$.

We can't find the limit by Substituting $x=4$.

We must first convert limit to an equivalent limit that is not indeterminate.

$$\lim_{x\to 4} \frac{x^2-4x}{x^2-3x-4} = \lim_{x\to 4} \frac{x(x-4)}{(x+1)(x-4)}$$

$\frac{0}{0}$ indeterminate form

Since $x \to 4$ we know $x \neq 4$, so $x-4 \neq 0$, so we can cancel $\frac{x-4}{x-4}$

$$= \lim_{x\to 4} \frac{x}{x+1}$$

limit of a rational function with $x=4$ in its domain. (not indeterminate)

Direct Substitution Property

$$= \frac{(4)}{(4)+1}$$

$$= \frac{4}{5}$$

Review: for $f(x) = \frac{x(x-4)}{x^2-3x-4}$, we have found $f(4)$ DNE

$\lim_{x\to 4} f(x) = \frac{4}{5}$

Graph is heading for location $(4, \frac{4}{5})$ But there is no point there. (hole!)

[Example 2] find $\lim\limits_{x \to -11} \dfrac{5x+55}{|x+11|}$

Experiment: what happen if we sub $x = -11$?

$$\frac{5(-11)+55}{|(-11)+11|} = \frac{-55+55}{|0|} = \frac{0}{0}$$

Indeterminate form! Can't just substitute in $x = 11$!

Review Absolute value Function   Piecewise-defined function

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

so $$|x+11| = \begin{cases} x+11 & \text{if } x+11 \geq 0 \\ -(x+11) & \text{if } x+11 < 0 \end{cases} = \begin{cases} x+11 & \text{if } x \geq -11 \\ -(x+11) & \text{if } x < -11 \end{cases}$$

In other words,

when $x \geq -11$ use formula $x+11$

when $x < -11$ use formula $-(x+11)$

So must do left limit and right limit separately.

Right Limit

use appropriate formula

$$\lim_{x\to -11^+} \frac{5x+55}{|x+11|} = \lim_{x\to -11^+} \frac{5x+55}{x+11}$$ indeterminate

$$= \lim_{x\to -11^+} \frac{5(x+11)}{(x+11)}$$ indeterminate

Since $x \to -11$, we know $x \neq -11$, so $x+11 \neq 0$, so we can cancel $\frac{x+11}{x+11}$

$$= \lim_{x\to -11^+} 5$$ no longer indeterminate

use limit law 7

$$= 5$$

So graph is approaching $(-11, 5)$ from the right

Left Limit

put in appropriate formula

$$\lim_{x \to -11^-} \frac{5x+55}{|x+11|} = \lim_{x \to -11^-} \frac{5x+55}{-(x+11)}$$

$$= \lim_{x \to -11^-} \frac{5(x+11)}{-(x+11)}$$

$$= \lim_{x \to -11^-} -5\frac{(x+11)}{(x+11)}$$

indeterminate

Since $x \to -11^-$ we know $x \neq -11$, so $x+11 \neq 0$ so we can cancel $\frac{x+11}{x+11}$

$$= \lim_{x \to -11^-} -5$$

not indeterminate

Use Limit Law 7

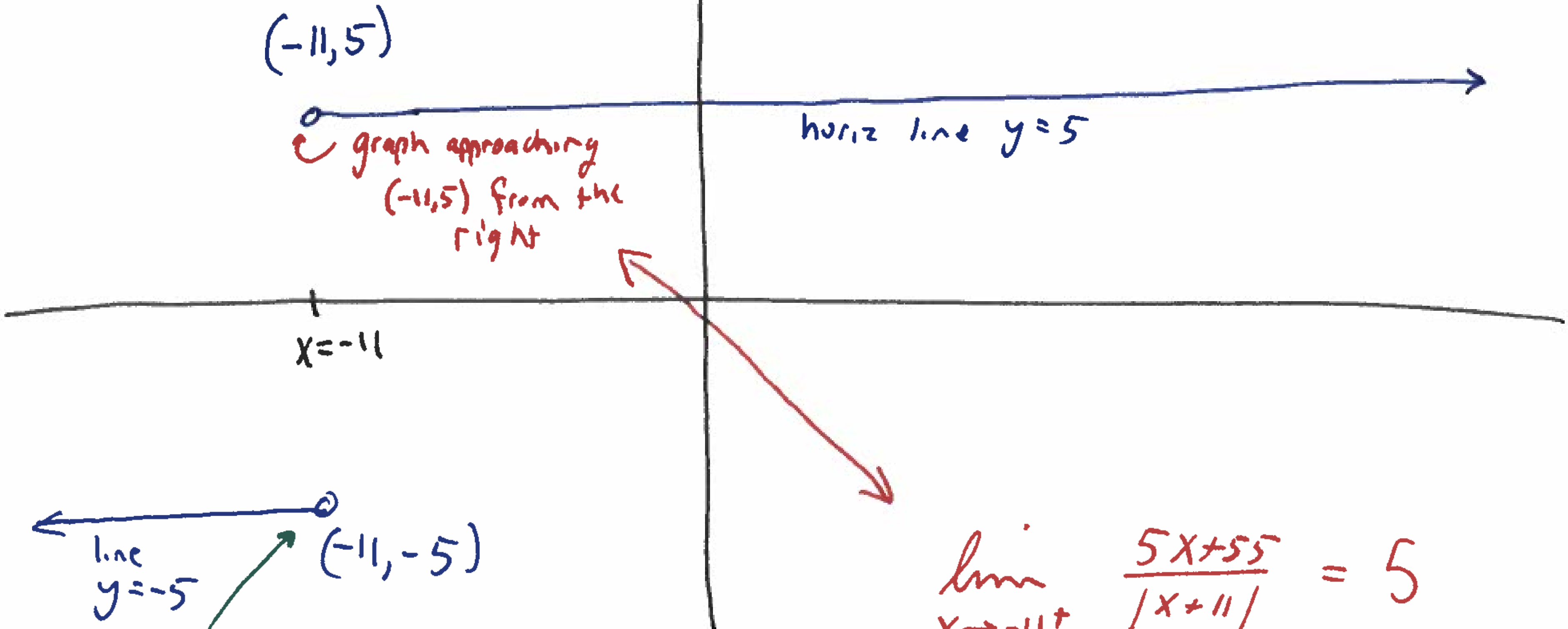
$$= -5$$

So graph is approaching $(-11,-5)$ from the left

$\lim_{x \to -11} \frac{5x+55}{|x+11|}$ DNE because left + right limits don't match.

Illustrate with graph

$$y = \frac{5x+55}{|x+11|}$$



$$\lim_{x \to -11^+} \frac{5x+55}{|x+11|} = 5$$

$$\lim_{x \to -11^-} \frac{5x+55}{|x+11|} = -5$$

[Example #3]

Observe $\frac{0}{0}$ indeterminate form

$$\lim_{x\to -5} \frac{\frac{1}{5}+\frac{1}{x}}{5+x} = \lim_{x\to -5} \frac{1}{(x+5)}\left(\frac{1}{5}+\frac{1}{x}\right)$$

get single base line

$$= \lim_{x\to -5} \frac{1}{x+5}\left(\frac{1(x)}{5(x)}+\frac{1(5)}{x(5)}\right)$$

getting common denominator

$$= \lim_{x\to -5} \frac{1}{(x+5)}\frac{(x+5)}{5x}$$

indeterminate

Since $x\to -5$, we know $x\neq -5$, so $x+5\neq 0$, so we can cancel $\frac{x+5}{x+5}$

$$= \lim_{x\to -5} \frac{1}{5x}$$ not indeterminate

rational function with $x=-5$ in domain

direct sub property

$$= \frac{1}{5(-5)}$$

$$= \frac{-1}{25}$$

[Example #4] involving a trick

$$\lim_{h\to 0} \frac{\sqrt{49+h}-7}{h} \overset{\text{trick}}{=} \lim_{h\to 0} \frac{\sqrt{49+h}-7}{h}\cdot\frac{(\sqrt{49+h}+7)}{(\sqrt{49+h}+7)}$$

indeterminate form

Still indeterminate

multiply by this special expression that is actually just 1

$$= \lim_{h\to 0} \frac{\sqrt{49+h}\sqrt{49+h} \cancel{-7\sqrt{49+h}} \cancel{+\sqrt{49+h}\cdot 7} - 7\cdot 7}{h(\sqrt{49+h}+7)}$$

$$= \lim_{h\to 0} \frac{\cancel{49}+h\cancel{-49}}{h(\sqrt{49+h}+7)}$$

Still indeterminate

$$= \lim_{h\to 0} \frac{h}{h(\sqrt{49+h}+7)}$$

Still indeterminate

Since $h\to 0$, we know $h\neq 0$, so we can cancel $\frac{h}{h}$

$$= \lim_{h\to 0} \frac{1}{\sqrt{49+h}+7}$$

No longer indeterminate

use limit law 5

$$\lim_{h \to 0} \frac{\sqrt{49+h} - 7}{h} = \frac{1}{\sqrt{49+0} + 7}$$

$$= \frac{1}{\sqrt{49} + 7}$$

$$= \frac{1}{7+7}$$

$$= \frac{1}{14}$$

So graph of $\frac{\sqrt{49+h}-7}{h}$ is heading for the location

$$(h,y) = \left(0, \frac{1}{14}\right)$$

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end of [Example 4]

End of Meeting
