MATH 2301 (Barsamian) Lecture #4 (meeting #6) Wed Sep 6, 2023

Pick up two handouts on the Intermediate Value theorem

Sign In

I still recommend buying a print copy of the textbook at College Bookstore uptown. (Cheap, for a printed book!)

Free binders in 2nd floor lobby of Morton Hall

use for written Exercises & textbook!

SI starts tomorrow (Thursday)

- Look for email from SI Leader Jake McCarthy
- go to Academic Achievement Center (AAC) website

Looking for one-on-one help with calculus

- Recitation Instructor office hours
- Peer tutoring at AAC
- I will have office hours starting in a week or so.

Quiz Q1 on Friday

Today: Section 1.5 Continuity

Definition of Continuity

words: f is continuous at a

usage: f is a function and a is a real number

meaning: This equation is true

$$\lim_{x \to a} f(x) = f(a)$$

This is equivalent to the following

Three Part Test for Continuity

Test 1 $\lim_{x \to a} f(x)$ exists

Test 1a: $\lim_{x \to a^-} f(x)$ exists

Test 1b: $\lim_{x \to a^+} f(x)$ exists

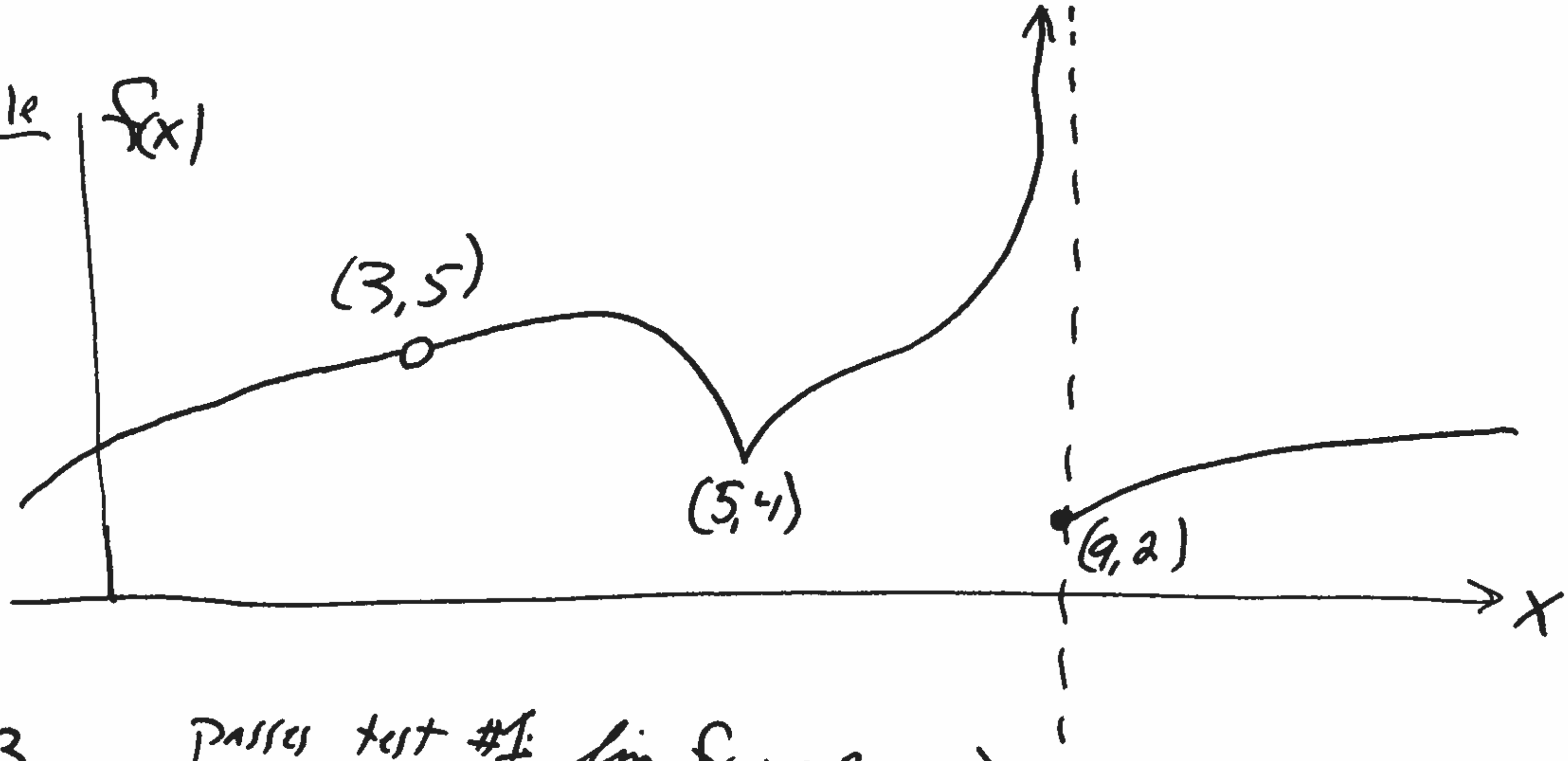
Test 1c: The left and right limits match

Test 2: The y value $f(a)$ exists

Test 3: The numbers from test 1 + test 2 match

$$\lim_{x \to a} f(x) = f(a)$$

Example



At $x=3$ passes test #1: $\lim_{x\to 3} f(x) = 2$

fails test #2 $f(3)$ DNE

(so also fails test #3)

f not continuous at $x=3$

At $x=5$ f is continuous because passes all three tests

at $x=9$

Test 1a $\lim_{x\to 9^-} f(x)$ DNE

Test 1b $\lim_{x\to 9^+} f(x) = 2$

Test 1c ~~lim f(x)~~ 1a & 1b don't match

fail test 1

Test 2 $f(9)=2$ passes test 2

Fails test 3

f not continuous at $x=9$

There is something nice at $x=9$

$\lim_{x \to 9^+} f(x) = 2$ graph heading for (9,2) from the right

$f(9) = 2$ point on graph at (9,2)

So $\boxed{\lim_{x \to 9^+} f(x) = f(9)}$

f is continuous from the right at $x=9$

See book for complete definition

Theorem

Polynomials are continuous at all x values

Rational functions are continuous everywhere on their domain.
That is, continuous at all x values except the x values that cause denominator = 0.

~~So polynomials that~~

That is, for all polynomials, at all x values, and for all rational functions, at all x values except x values that cause denominator = 0

$$\lim_{x \to a} f(x) = f(a)$$

is a true equation

This is true because of the Direct Substitution Property for Limits

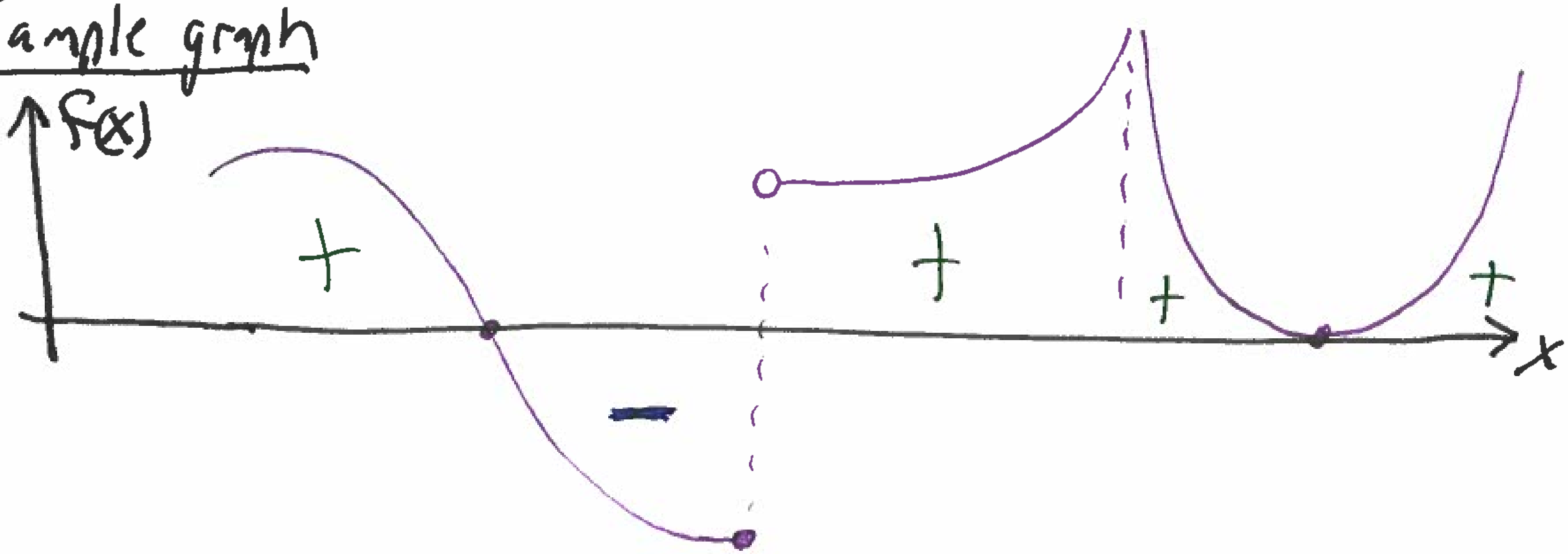
Later in 2301, we will want to investigate

Sign Behavior of a function $f(x)$

that is where is $f(x)$ positive? where is $f(x)$ negative?

That is where is $f(x) > 0$, where is $f(x) < 0$?

Sample graph



The only x values where a function $f(x)$ can change sign are

- x values where $f(x) = 0$. (x intercept at $(a, 0)$)
- x values where f is discontinuous.

Another topic involving continuity

The Intermediate Value Theorem

See handout

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Using The Intermediate Value Theorem

The Theorem as stated in many books. (Similar, but not identical, to the statement in our book):

> **The Intermediate Value Theorem**
>
> Suppose that f is continuous on the closed interval $[a, b]$ and such that ~~where~~ $f(a) \neq f(b)$.
>
> Then for any real number y between $f(a)$ and $f(b)$,
>
> there exists some number c in (a, b) such that $f(c) = \cancel{N}$ y

Presented another way:

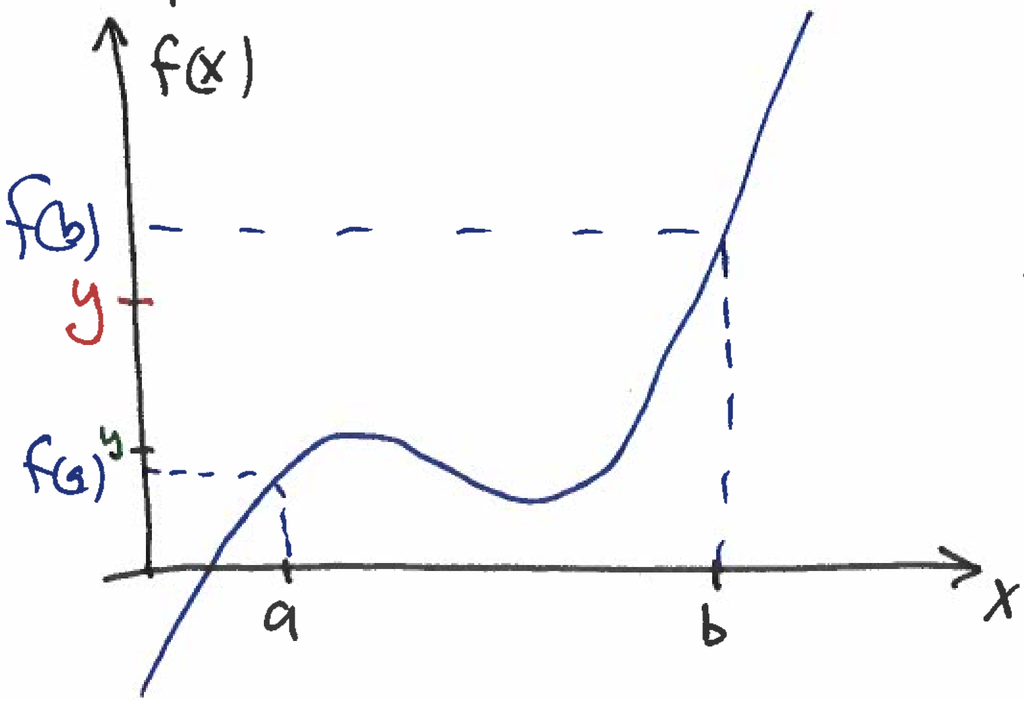
> **The Intermediate Value Theorem**
>
> If we know that all this stuff is true (these are the "hypotheses"):
>
> - There is a closed interval $[a, b]$.
> - The function f is continuous on the closed interval $[a, b]$.
> - The numbers $f(a) \neq f(b)$.
> - The number y is a real number between $f(a)$ and $f(b)$.
>
> Then we are allowed to say these words (this is the "conclusion"):
>
> - "There exists at least one number c in (a, b) such that $f(c) = y$."

When we "use" this theorem (or any other), we start by verifying explicitly that all of the hypotheses are satisfied in our specific situation. If they are satisfied, then we write the conclusion, adapted to our specific situation.

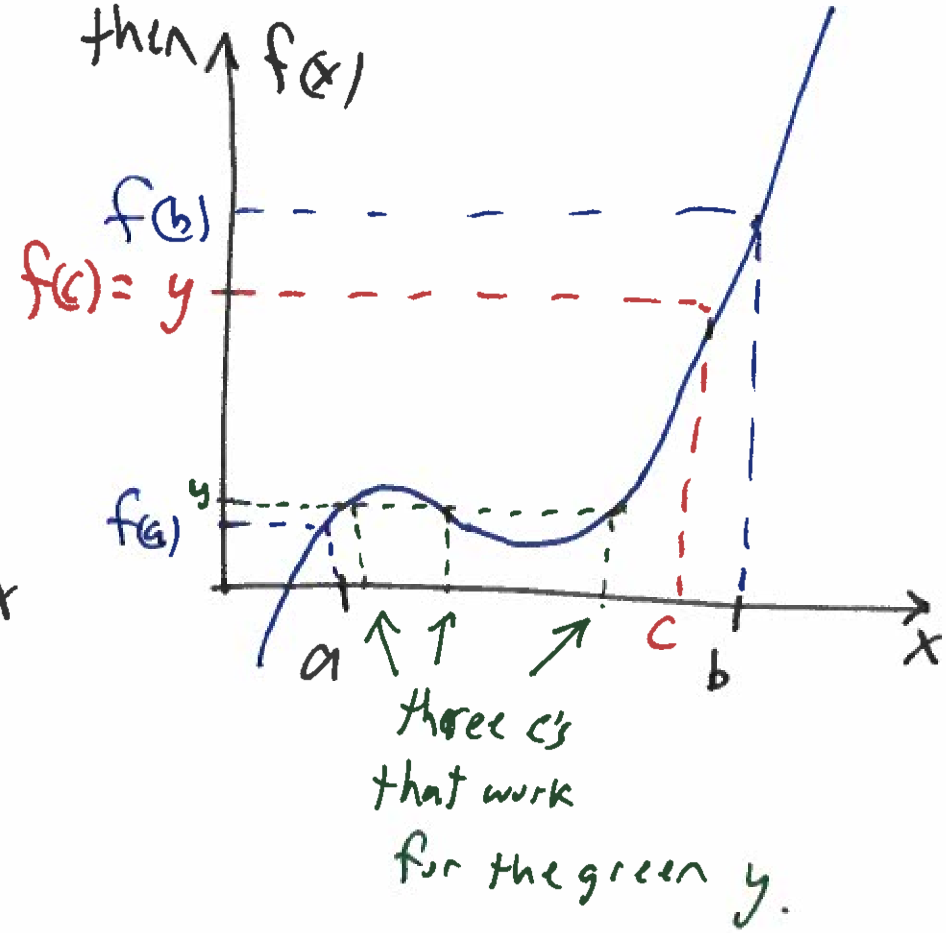
A worksheet for this process is provided on the back of this page.

Picture of the Statement of the Intermediate Value Theorem

If



then



[Example] of <u>using</u> the Intermediate value theorem

Show that there is a <u>root</u> of the equation

$$17x^{13} - 15x^{12} + 235x^5 - 13x^2 + 5 = 0$$

(show that there is an x value that makes the equation true.)

let $f(x) = 17x^{13} - 15x^{12} + 235x^5 - 13x^2 + 5$

make up interval $[-10, 10]$

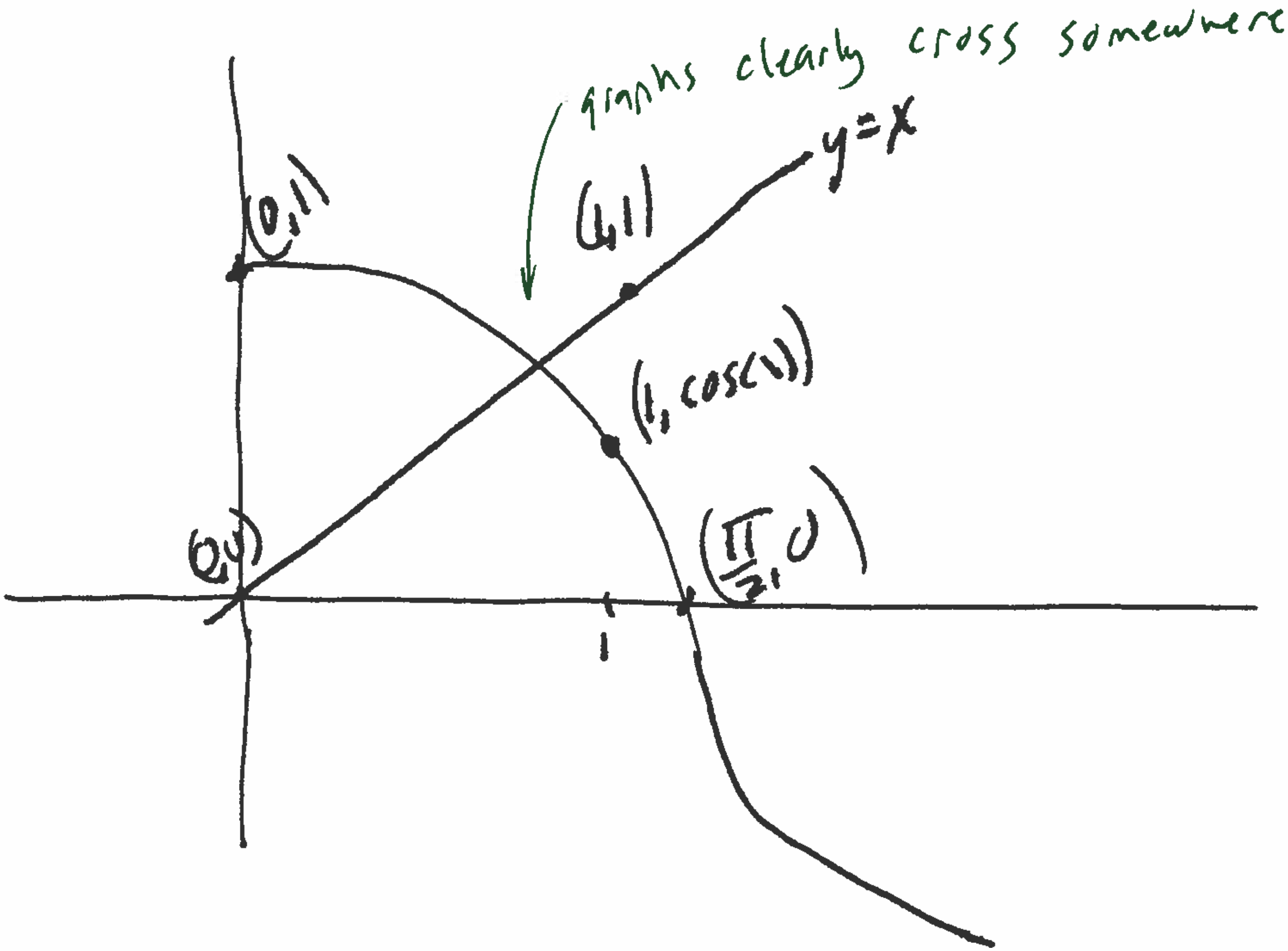
Observe: $f(-10)$ = huge, negative number
$f(10)$ = huge, positive

Intermediate Value Theorem Worksheets

Generic Hypotheses	Our Specific Hypotheses
the closed interval $[a, b]$	$[-10, 10]$
the function $f(x)$	$f(x) = 17x^{13} - 15x^{12} + 235x^5 - 13x^2 + 5$
verification that f is continuous on the closed interval $[a, b]$.	f is polynomial, so continuous
the value of $f(a)$	$f(a) = f(-10) =$ huge negative number
the value of $f(b)$	$f(b) = f(10) =$ huge positive number
confirmation that $f(a) \neq f(b)$	$f(a) \neq f(b)$ ✓
the real number "y"	let $y = 0$
verification that y is between $f(a)$ and $f(b)$	$f(a) < 0 < f(b)$ ✓
Generic Conclusion	**Our Specific Conclusion**
"There exists at least one number c in the open interval (a, b) such that $f(c) = y$."	"There exists at least one number c in the open interval $(-10, 10)$ such that $f(c) =$ 0."

Generic Hypotheses	Our Specific Hypotheses
the closed interval $[a, b]$	
the function $f(x)$	
verification that f is continuous on the closed interval $[a, b]$.	
the value of $f(a)$	
the value of $f(b)$	
confirmation that $f(a) \neq f(b)$	
the real number "y"	
verification that y is between $f(a)$ and $f(b)$	
Generic Conclusion	**Our Specific Conclusion**
"There exists at least one number c in the open interval (a, b) such that $f(c) = y$."	"There exists at least one number c in the open interval ____ such that $f(c) =$ _____."

[Example] Show that there is a root of the equation $\cos(x) = x$ in the interval $(0,1)$



To use intermed value theorem

let interval $[0,1]$

let $f(x) = \cos(x) - x$

show there is an x in interval $(0,1)$

where $\cos(x) - x = 0$

observe $f(0) = \cos(0) - 0 = 1 - 0 = 1$

$f(1) = \cos(1) - 1 =$ negative number

notice that

$\cos(1) < 1$

Intermediate Value Theorem Worksheets

Generic Hypotheses	Our Specific Hypotheses
the closed interval $[a, b]$	$[0,1]$
the function $f(x)$	$f(x) = \cos(x) - x$
verification that f is continuous on the closed interval $[a, b]$.	continuous ✓
the value of $f(a)$	$f(0) = \cos(0) - 0 = 1 - 0 = 1$
the value of $f(b)$	$f(1) = \cos(1) - 1 =$ neg
confirmation that $f(a) \neq f(b)$	$f(0) \neq f(1)$
the real number "y"	let $y = 0$
verification that y is between $f(a)$ and $f(b)$	$f(0) = 1 > 0 > f(1) =$ neg
Generic Conclusion	**Our Specific Conclusion**
"There exists at least one number c in the open interval (a, b) such that $f(c) = y$."	"There exists at least one number c in the open interval $(0,1)$ such that $f(c) =$ 0."

Generic Hypotheses	Our Specific Hypotheses
the closed interval $[a, b]$	
the function $f(x)$	
verification that f is continuous on the closed interval $[a, b]$.	
the value of $f(a)$	
the value of $f(b)$	
confirmation that $f(a) \neq f(b)$	
the real number "y"	
verification that y is between $f(a)$ and $f(b)$	
Generic Conclusion	**Our Specific Conclusion**
"There exists at least one number c in the open interval (a, b) such that $f(c) = y$."	"There exists at least one number c in the open interval ____ such that $f(c) =$ ____."

Intermediate Value Theorem tells us that

for $f(x) = \cos(x) - x$

there exists an x value "c" with $0 < c < 1$

Such that $f(c) = 0$.

that is $\cos(c) - c = 0$

That means that $\cos(c) = c$

So we have proven that there is an x value "c" in the interval $(0,1)$ that makes this equation true:

$$\cos(x) = x.$$

So "c" is a root of the equation, and "c" is in the interval $(0,1)$

end of lecture