MATH 2301 Lecture #5 (meeting #7) Fri Sep 8, 2023

Pick up any handouts that you don't already have
Sit in Alternate seats (see seating chart)
Sign In
Quiz Q1 Today at end of class

Remember to buy a print copy of the Book at College Bookstore
if you like having a printed copy.
Remember to pick up a free 3-Ring Binder in 2nd floor lobby.
Use it for your Written Homework
Recitation Assignments for Tue Sep 12 are posted on the Course Webpage

Today + Monday Section 1.6 Limits Involving Infinity

Today: "Infinite Limits"

Definition of Infinite Limit

Symbol: $\lim_{x \to a} f(x) = \infty$

Spoken: The limit, as x approaches a, of $f(x)$ is infinity.

Usage: f is a function, "a" is a real number

Meaning: The value of $f(x)$ can be made arbitrarily large and positive (as large a positive as desired) by making x sufficiently close to a, but not equal to a.

Straightforward variation (six of them) such as

$\lim_{x \to a^-} f(x) = \infty$ or $\lim_{x \to a} f(x) = -\infty$

Definition of Vertical asymptote

Words: The line $x=a$ is a vertical asymptote of $f(x)$

Meaning: At least one of these limit expressions is true

$$\lim_{x\to a} f(x) = \infty$$

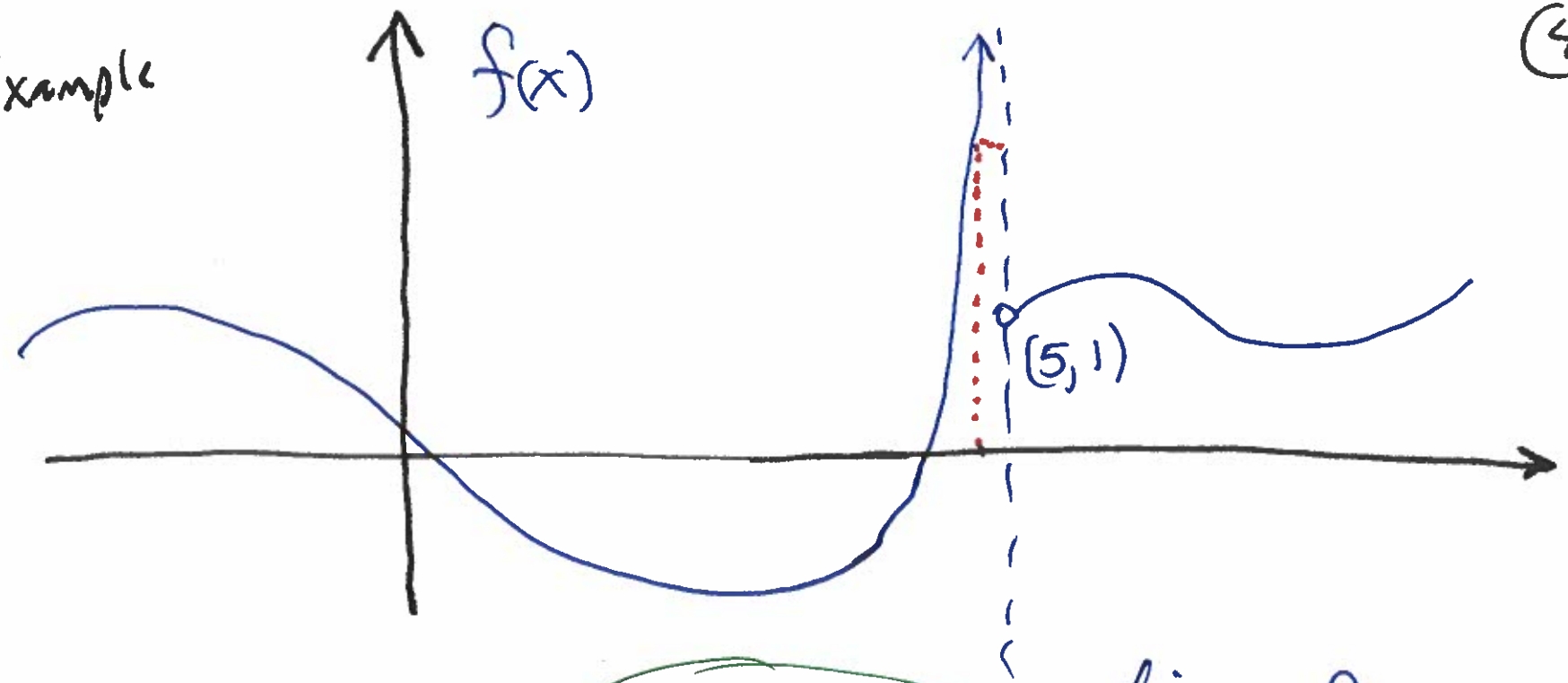
$$\lim_{x\to a^-} f(x) = \infty$$

$$\vdots$$

$$\lim_{x\to a} f(x) = -\infty$$

Six variations of infinite limits

Example



$\lim_{x \to 5^-} f(x) = \infty$

$\lim_{x \to 5^+} f(x) = 1$

$\lim_{x \to 5} f(x)$ DNE

The line $x = 5$ is a vertical asymptote for $f(x)$.

[Example 1] We are interested in this limit

$$\lim_{x\to 4} \frac{x^2-3x-2}{x-4} = \lim_{x\to 4} \frac{(x-1)(x-2)}{x-4}$$

(a) Estimate limit using tables of values

x	$f(x)$
4.1	$f(4.1) = \frac{(4.1-1)(4.1-2)}{(4.1-4)} = \frac{(3.1)(2.1)}{0.1} = \frac{\text{close to } 6}{1/10} = \text{close to } 60$
4.01	$f(4.01) = \frac{(4.01-1)(4.01-2)}{4.01-4} = \frac{(3.01)(2.01)}{0.01} = \frac{\text{close to } 6}{1/100} = \text{close to } 600$
4.001	$f(4.001) = \cdots = \frac{(3.001)(2.001)}{0.001} = \frac{\text{close to } 6}{1/1000} = \text{close to } 6000$

trend $x \to 4^+$

y values getting more & more positive without bound

$$\lim_{x\to 4^+} f(x) = \infty.$$

x	$f(x) = \dfrac{(x-1)(x-2)}{(x-4)}$
3.9	$f(3.9) = \dfrac{(3.9-1)(3.9-2)}{3.9-4} = \dfrac{(2.9)(1.9)}{-0.1} = \dfrac{\text{close to } 6}{-1/10} = \text{close to } -60$
3.99	$f(3.99) = \dfrac{\cdots}{(3.99-4)} = \dfrac{(2.99)(1.99)}{-0.01} = \dfrac{\text{close to } 6}{-1/100} = \text{close to } -600$
3.999	$f(3.999) = \cdots = \dfrac{\text{close to } 6}{-1/1000} = \text{close to } -6000$

trend $x \to 4^-$

trend

y getting more & more negative without bound.

$$\lim_{x \to 4^-} f(x) = -\infty$$

Conclusion: $\lim_{x \to 4} f(x)$ Does not exist because left & right limits dont match.

Observe using ideas of Section 1.4

limit of numerator $= \lim_{x \to 4} (x-1)(x-3) = ((4)-1)((4)-3) = 3 \cdot 2 = 6$

limit of denominator $= \lim_{x \to 4} x-4 = (4)-4 = 0$

Missing theorem from section 1.4 would say

$\lim_{x \to 4^-} f(x)$ DNE

$\lim_{x \to 4^+} f(x)$ DNE

$\lim_{x \to 4} f(x)$ DNE

} using definition of Limit from Section 1.3 and 1.4

So we have <u>expanded</u> the definition of <u>Limit</u>

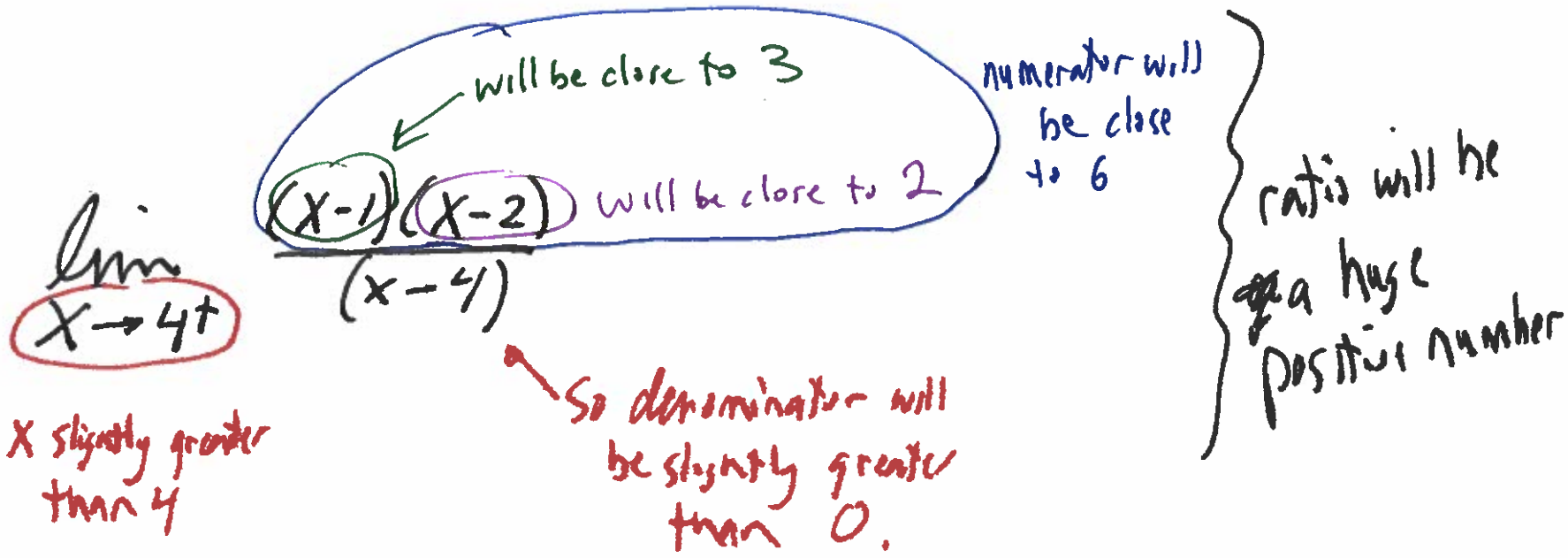
$\lim_{x \to 4^-} f(x) = -\infty$

$\lim_{x \to 4^+} f(x) = +\infty$

$\lim_{x \to 4} f(x)$ DNE because left and right limits dont match

} Using Definition of limit from Section 1.6 (<u>expanded</u> definition, incorporating the concept of <u>infinity</u>)

Method of analyzing relative size of numerator + denom



The closer x gets to 4 from the right, the more positive $f(x)$ gets without bound

$$\lim_{x \to 4^+} f(x) = +\infty$$

Similarly

$\lim_{x \to 4^-}$

x slightly less than 4

$(x-1)(x-2)$ ← close to 2, close to 3

numerator close to 6

$x-4$ denom slightly less than 0

rate will be huge neg number

$$\lim_{x \to 4^-} f(x) = -\infty$$

~~Error of Limit~~ So $\lim_{x \to 4} f(x)$ DNE because left + right limits dont match

End of meeting