MATH 2301 (Barsamian) Lecture #7, Wed Sep 13, 2023

Pick up new handout on Table

Sign In

Remember: Friday Quiz Q2 (covering sections 1.5 and 1.6)

Recitation on Tue Sep 19: Come Prepared

College Bookstore still has print copies of the book available at a great price.
(At some point, unsold stock will be sent back to the publisher)

Free 3-ring binders in Morton 2nd Floor Lobby

When you email me, or your Recitation Instructor, please put Regarding MATH 2301 Section _____ in the subject line

Today: Discussing Section 2.1

Rates of Change,

Secant Lines

Tangent Lines

All definitions on the new handout.

(only some definitions are in Section 2.1)

Rates of Change and Secant and Tangent Lines (Concepts from Section 2.1)

Definition of Average Rate of Change

Words: Average Rate of Change of f from a to b

Usage: a, b are real numbers, $a < b$, and f is a function that is continuous on the interval $[a, b]$.

Meaning: the number $m = \frac{f(b)-f(a)}{b-a}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(b, f(b))$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the average rate of change is called the *average velocity* from time a to time b.

Alternate presentation of average rate of change:

Words: Average Rate of Change of f from a to $a + h$

Usage: a, h are real numbers, $h \neq 0$, and f is a function that is continuous on an interval near a

Meaning: the number $m = \frac{f(a+h)-f(a)}{h}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(a + h, f(a + h))$

Definition of Instantaneous of Change

Words: Instantaneous Rate of Change of f at a

Symbol: $f'(a)$

Spoken: The derivative of f at a

Usage: a is a real number and f is a function that is continuous near $x = a$

Meaning: the number $m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the Instantaneous rate of change is called the *instantaneous velocity* at time a

Definition of line tangent to graph of f at $x = a$

The line that has these two properties

- contains the *point* $(a, f(a))$ (This point is called the *point of tangency*.)
- has *slope* $m = f'(a)$ (This number is called the *slope of the tangent line* at $x = a$, but it is also called the *slope of the graph* of $f(x)$ at $x = a$.)

General Point Slope Form of the Equation of the Tangent Line

The line tangent to the graph of $f(x)$ at $x = a$ has equation

$$(y - f(a)) = f'(a)(x - a)$$

Today: Extended Example

An object moves in 1 dimension with position function

$$f(t) = 6t - t^2$$

where t is time in seconds

and $f(t)$ is the position, in meters, at time t seconds.

ⓐ Find the Average velocity from time $t=1$ to $t=4$.

Solution: we need to find $m = \dfrac{f(4) - f(1)}{4-1} = \dfrac{f(4) - f(1)}{3}$

Get parts

$f(1) = 6(1) - (1)^2 = 6 - 1 = 5$ meters

$f(4) = 6(4) - (4)^2 = 24 - 16 = 8$ meters

So $m = \dfrac{8m - 5m}{4sec - 1sec} = \dfrac{3m}{3sec} = 1 \dfrac{m}{sec}$ average velocity

(b) Illustrate result of (a) using a graph of $f(t)$

<u>Solution</u> graph $f(t)$, draw secant line, label its slope.

$$f(t) = 6t - t^2 = t(6-t)$$

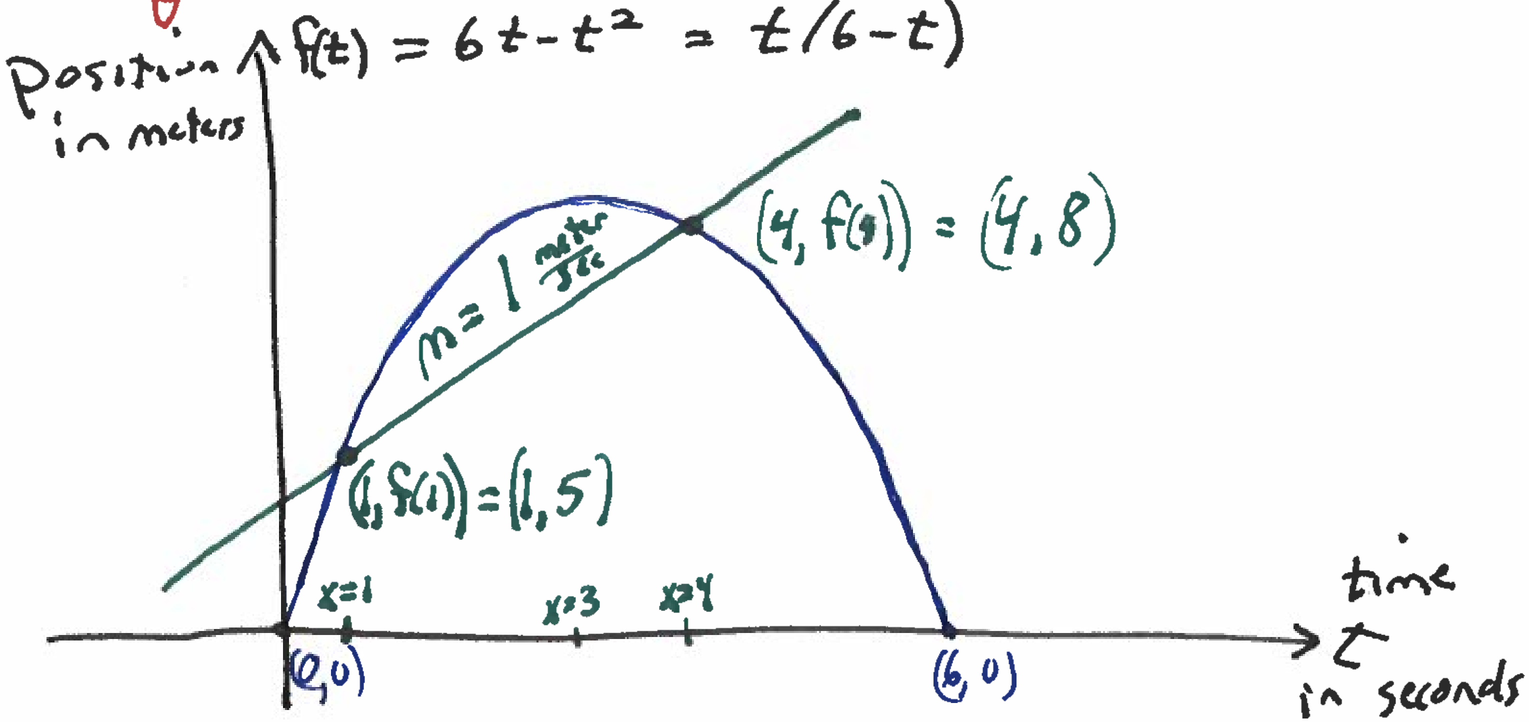
Standard form — factored form

horiz axis intercepts at $(0,0)$ and $(6,0)$

$t^2 \Rightarrow$ parabola

$-t^2 \Rightarrow$ parabola facing down

no constant term $\Rightarrow$ y intercept at $(t,y) = (0,0)$



c) Find average velocity from time $t=1$ to time $t=1+h$, where $h=2$, $h=1$, $h=0.5$, $h=0.1$

Solution

We need to find

$$m = \frac{f(1+h) - f(1)}{(1+h) - 1} = \frac{f(1+h) - f(1)}{h}$$

We already have $f(1) = 5$

Build $f(1+h) = 6(1+h) - (1+h)^2$

replace t with $1+h$

$$= 6 + 6h - \left((1+h)(1+h)\right)$$

$$= 6 + 6h - \left(1 + 2h + h^2\right)$$

$$= 6 + 6h - 1 - 2h - h^2$$

$$= 5 + 4h - h^2$$

Now build m

$$m = \frac{f(1+h) - f(1)}{h} = \frac{(\cancel{5} + 4h - h^2) - \cancel{5}}{h} = \frac{4h - h^2}{h} = \frac{h(4-h)}{h}$$

Since all the h values we will be using are non-zero, we can cancel $\frac{h}{h}$

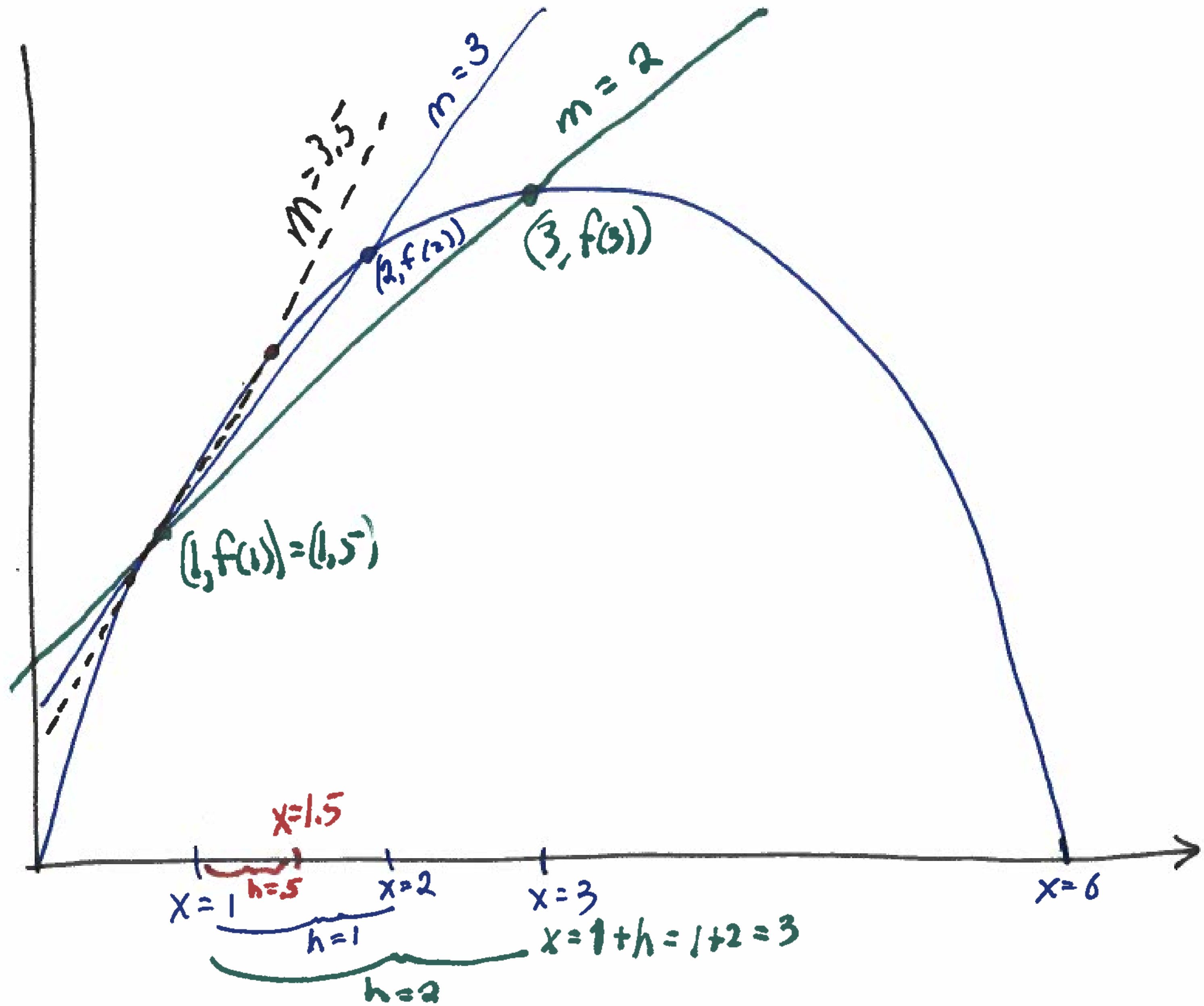
$$m = 4 - h$$

Make Table of Values

h	m = 4 - h
2	m = 4 - 2 = 2
1	m = 4 - 1 = 3
0.5	m = 4 - 0.5 = 3.5
0.1	m = 4 - 0.1 = 3.9

these are the average velocities

(d) Illustrate results of (c) with a graph



c) Find the Instantaneous Velocity at $t=1$

Solution: We need to find this

$$m = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} \quad \frac{\text{meters}}{\text{seconds}}$$

all the steps from page 7 ~~work~~ work, including the cancelling because $h \neq 0$

$$= \lim_{h \to 0} 4 - h$$

limit of a polynomial

Can use direct substitution property, sub in $h=0$

$$= 4 - (0)$$

$$\boxed{m = 4 \ \frac{\text{meters}}{\text{second}}}$$

f) Find slope of the line tangent to graph of $f(t)$ at $t = 1$.

Solution: we need to find

$$m = f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \ldots = 4 \ \frac{\text{meters}}{\text{second}}$$

using result of (e)

(g) Find the equation of the line tangent to graph of $f(t)$ at $t=1$

Solution: we need to build this equation

$$(y - f(a)) = f'(a)(t-a)$$

General Point Slope Form of the Equation of the Tangent Line

Get Parts

$a = 1$ (This is the t coordinate of the point of tangency)

$f(a) = f(1) = 5$ (this is the y coordinate of the point of tangency)

from earlier (see page 4)

$f'(a) = f'(1) = 4$ (this is the slope of the tangent line)

from part (f), page 10

Now Put the Parts into the Equation

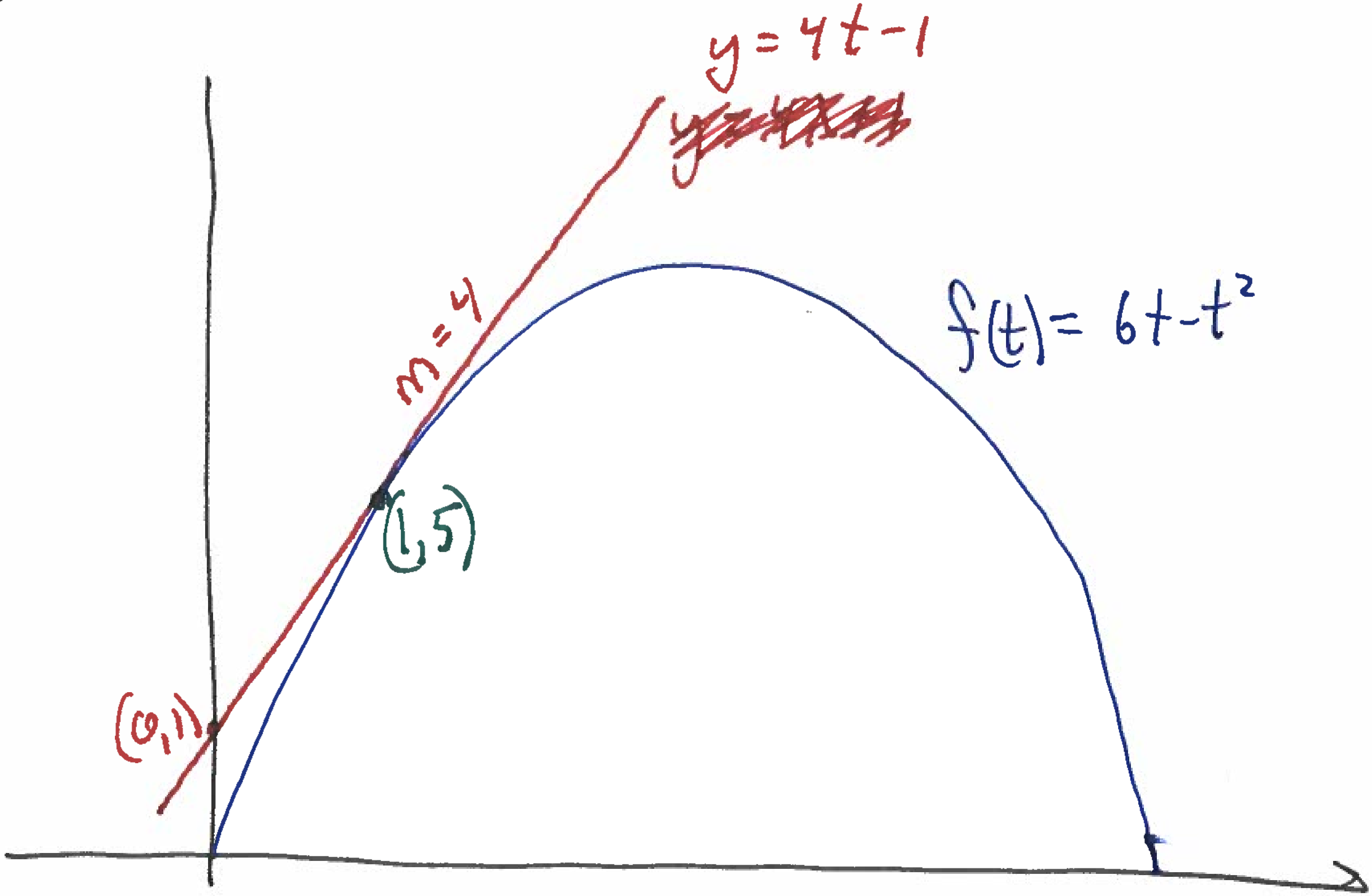
$(y-5) = 4(t-1)$ point slope form of the equation of the tangent line

Convert to Slope Intercept form by solving for y

$y - 5 = 4t - 4$

$y = 4t + 1$ Slope intercept form of the equation of the tangent line

(h) Illustrate with graph



end of meeting