MATH 2301 (Barsamian) Lecture #8, Fri Sep 15, 2023

Pick up Graded Quiz

If you don't already have Handout on Rates of Change, Pick up one now

Alternate Seating

Sign In

Quiz Q2 Today

Recitation R4 on Tuesday Sep 19
Assignments have been posted

Reminder: Binders for free in 2nd Floor Lobby
Printed Textbooks for Cheap at College Bookstore

Rates of Change and Secant and Tangent Lines (Concepts from Section 2.1)

Definition of Average Rate of Change

Words: Average Rate of Change of f from a to b

Usage: a, b are real numbers, $a < b$, and f is a function that is continuous on the interval $[a, b]$.

Meaning: the number $m = \frac{f(b)-f(a)}{b-1}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(b, f(b))$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the average rate of change is called the *average velocity* from time a to time b.

Alternate presentation of average rate of change:

Words: Average Rate of Change of f from a to $a + h$

Usage: a, h are real numbers, $h \neq 0$, and f is a function that is continuous on an interval near a

Meaning: the number $m = \frac{f(a+h)-f(a)}{h}$

Graphical Significance: the number m is the slope of secant line that passes through points $(a, f(a))$ and $(a + h, f(a + h))$

Definition of Instantaneous of Change

Words: Instantaneous Rate of Change of f at a

Symbol: $f'(a)$

Spoken: The derivative of f at a

Usage: a is a real number and f is a function that is continuous near $x = a$

Meaning: the number $m = \lim_{h \to 0} \frac{f(a+h)-f(a)}{h}$

Additional terminology: When the variable is t, representing *time* and the function $f(t)$ is a *position function*, representing the *position* of an object at time t, then the Instantaneous rate of change is called the *instantaneous velocity* at time a

Definition of line tangent to graph of f at $x = a$

The line that has these two properties

- contains the *point* $(a, f(a))$ (This point is called the *point of tangency*.)
- has *slope* $m = f'(a)$ (This number is called the *slope of the tangent line* at $x = a$, but it is also called the *slope of the graph* of $f(x)$ at $x = a$.)

General Point Slope Form of the Equation of the Tangent Line

The line tangent to the graph of $f(x)$ at $x = a$ has equation

$$(y - f(a)) = f'(a)(x - a)$$

Today Continuing Section 2.1 Rates of Change, Tangent Lines

[Example #1] (Similar to 2.1 #5)

(a) Find equation of line tangent $y = \sqrt{x}$ at $x = 9$

Present equation in slope intercept form.

(b) Illustrate result of (a)

Solution

(a) We need to build $(y - f(a)) = f'(a)(x - a)$

Get Parts

$a = 9$ x coord of point of tangency

$f(a) = f(9) = \sqrt{9} = 3$ y coord of point of tangency

4

$$f'(a) = f'(9) = \lim_{h \to 0} \frac{f(9+h) - f(9)}{h}$$

indeterminate

$f(x) = \sqrt{x}$

$f(\ ) = \sqrt{(\ )}$ empty incision

$f(9+h) = \sqrt{9+h}$

$$= \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{9+h} - 3}{h} \cdot \frac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{\sqrt{9+h}\sqrt{9+h} \cancel{- 3\sqrt{9+h}} \cancel{+ \sqrt{9+h} \cdot 3} - 3 \cdot 3}{h(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{\cancel{9} + h \cancel{- 9}}{h(\sqrt{9+h} + 3)}$$

$$= \lim_{h \to 0} \frac{h}{h(\sqrt{9+h} + 3}$$

Still indeterminate

Since $h \to 0$, we know $h \neq 0$ so we can cancel $\frac{h}{h}$

$$= \lim_{h \to 0} \frac{1}{\sqrt{9+h} + 3}$$

no longer indeterminate!

$$= \frac{1}{\sqrt{9+(0)} + 3}$$

$$= \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{3+3}$$

$f'(a) = \frac{1}{6}$ = slope of tangent line

Substitute parts into equation

$$(y-3) = \frac{1}{6}(x-9)$$ point slope form

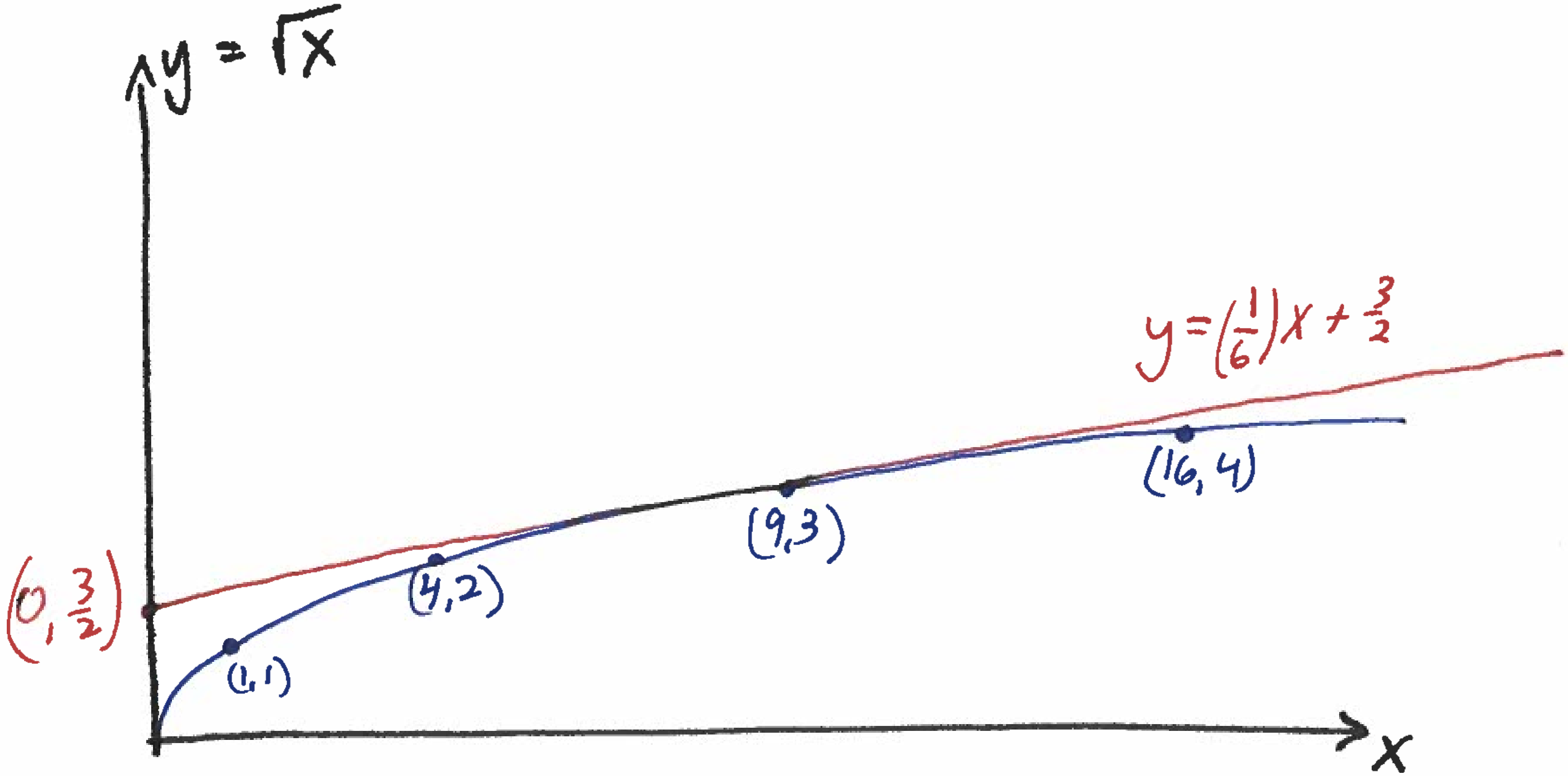
Convert to slope intercept form by solving for y

$$y - 3 = \left(\frac{1}{6}\right)x - \frac{9}{6} = \left(\frac{1}{6}\right)x - \frac{3}{2}$$

$$y = \left(\frac{1}{6}\right)x - \frac{3}{2} + 3$$

$$y = \left(\frac{1}{6}\right)x + \frac{3}{2}$$ slope intercept

(b) Illustrate with a graph

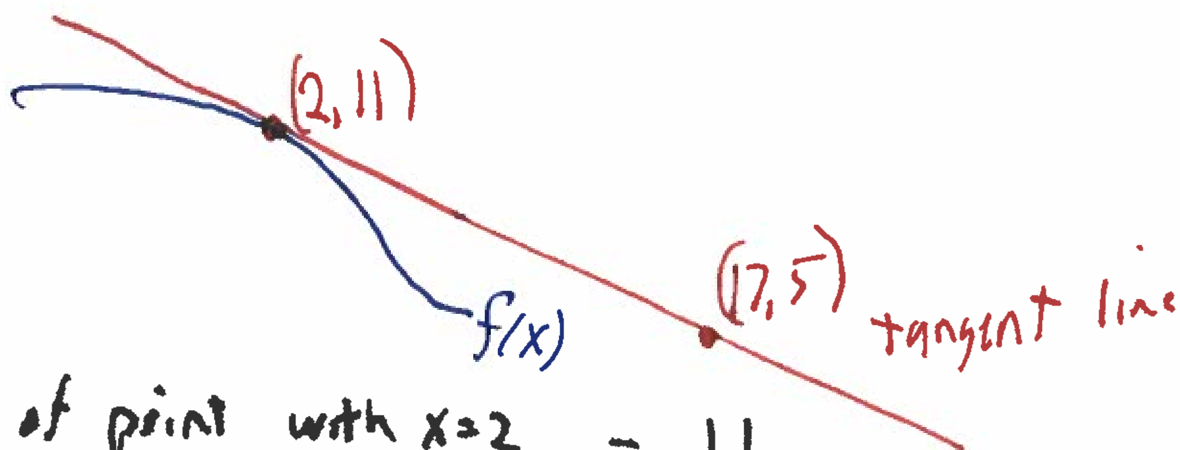


End of Example

Example

The line tangent to graph of $f(x)$ at $(2, 11)$ passes through the point $(17, 5)$. Find $f(2)$ and $f'(2)$

Solution



$f(2)$ = y coord of point with $x = 2$ $= 11$

$f(2) = 11$

$f'(2)$ = slope of the line that is tangent at $x=2$ $= m = \frac{\Delta y}{\Delta x} = \frac{5-11}{17-2} = \frac{-6}{15} =$

$= -\frac{2}{5}$

End of Example and End of lecture