MATH 2301 (Barsamian) Lecture 9 (Mon Sep 18, 2023) ①

Pick Up Handout

Sign In

Be sure to Prep for Recitation tomorrow (Tue Sep 19)

Exam X1 This coming Friday (Sep 22)

Review Policy for Absences & Make-ups on Course Web Page

Recall

Definition of Derivative of f at "a", where "a" is a real number

Symbol: $f'(a)$

meaning: the number $\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$ indeterminate form

This number is the slope m of the line tangent to graph of f at $x=a$.

Today: Start Discussing Section 2.2 The derivative as a function

Replace the number "a" with a variable, such as "x".
The result is a function of that variable

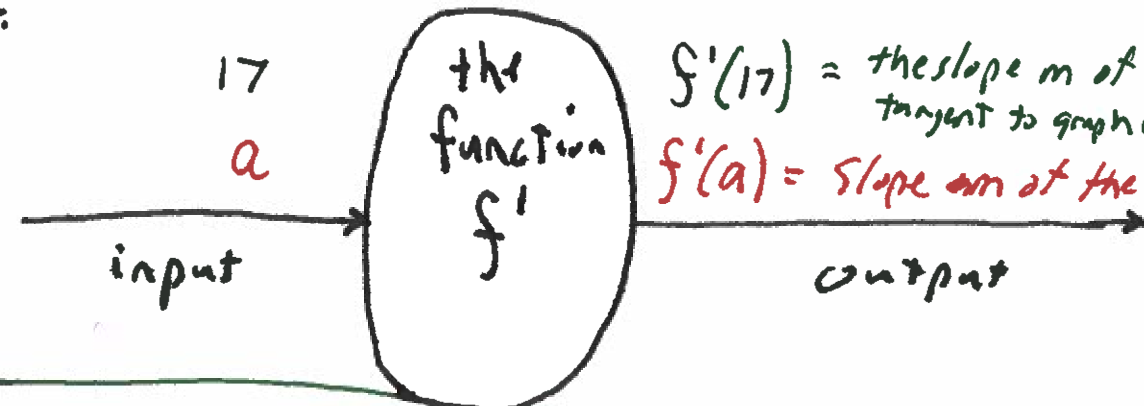
The Definition of the Derivative

words: The derivative of $f(x)$

symbol: $f'(x)$

meaning: The function $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ indeterminate form

machine diagram:



$f'(17)$ = the slope m of the line tangent to graph of f at x=17

$f'(a)$ = slope m of the line tangent to graph of f at x=a

[Example 1]

MATH 2301 (Barsamian) Class Activity: Find the Derivative of a Function Given by a Graph

Goal: Given the graph of f on the top axes on the next page, make a graph of f' on the bottom axes.

On the graph of f', the input will be x and the output will be $f'(x)$.

This means that when a ***particular real number*** $\boldsymbol{x = a}$ is used as *input* to the function $f'(x)$, the *resulting output* will be the **real number** $\boldsymbol{f'(a)}$.

Remember the graphical interpretation of $f'(a)$, where a is a particular ***real number***:

Definition of the *Derivative of f at a*

- **symbol:** $f'(a)$
- **graphical interpretation:** $f'(a)$ is the number that is the slope of the line tangent to the graph of f at the point on the graph where $x = a$.

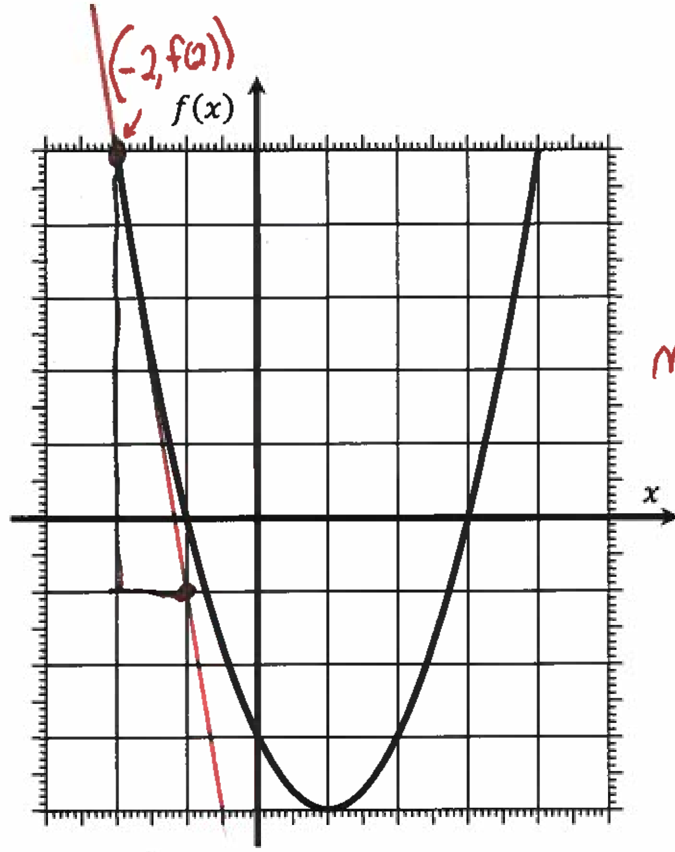
We build a graph of $f'(x)$ by making a table with particular real number values of x in the left column, to use as inputs. (These can be thought of as a bunch of different $x = a$ values) We then find the resulting real number values of $f'(x)$. (That is, the corresponding values of $f'(a)$.)

Part 1: Prepare the data for your graph of f' by filling out the following table.

x	what to do on the graph of f	$f'(x)$
-2	Draw the line tangent to the graph of f at the point where $x = -2$ and find its slope m. This slope m will be the value of $f'(-2)$.	-6
-1	Draw the line tangent to the graph of f at the point where $x = -1$ and find its slope m. This slope m will be the value of $f'(-1)$.	-4
0	Draw the line tangent to the graph of f at the point where $x = 0$ and find its slope m. This slope m will be the value of $f'(0)$.	-2
1	Draw the line tangent to the graph of f at the point where $x = 1$ and find its slope m. This slope m will be the value of $f'(1)$.	0
2	Draw the line tangent to the graph of f at the point where $x = 2$ and find its slope m. This slope m will be the value of $f'(2)$.	2
3	Draw the line tangent to the graph of f at the point where $x = 3$ and find its slope m. This slope m will be the value of $f'(3)$.	4
4	Draw the line tangent to the graph of f at the point where $x = 4$ and find its slope m. This slope m will be the value of $f'(4)$.	

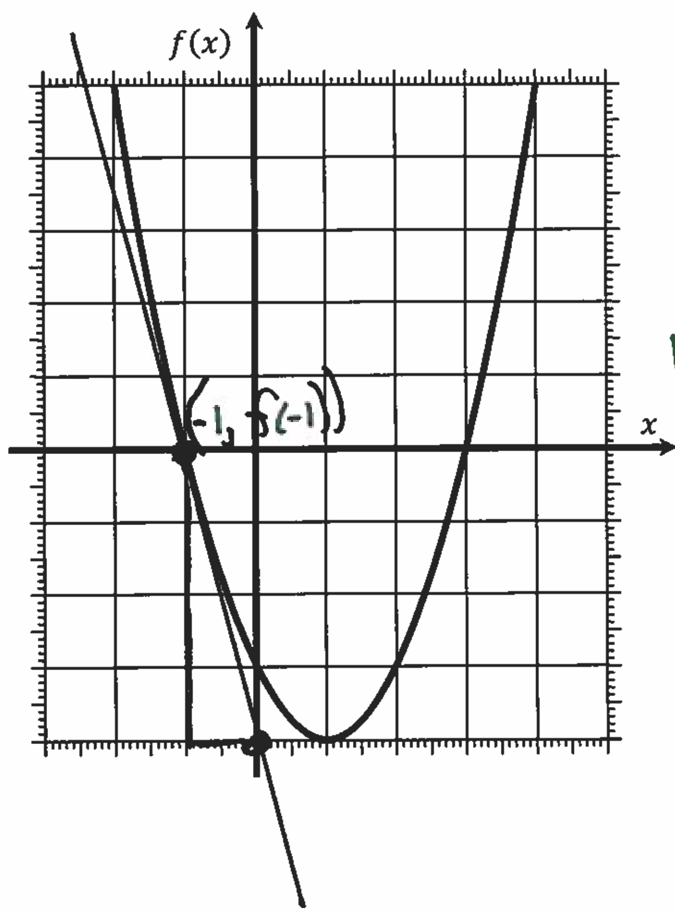
Part 2 is on back ➔

tangent line at $x=-2$



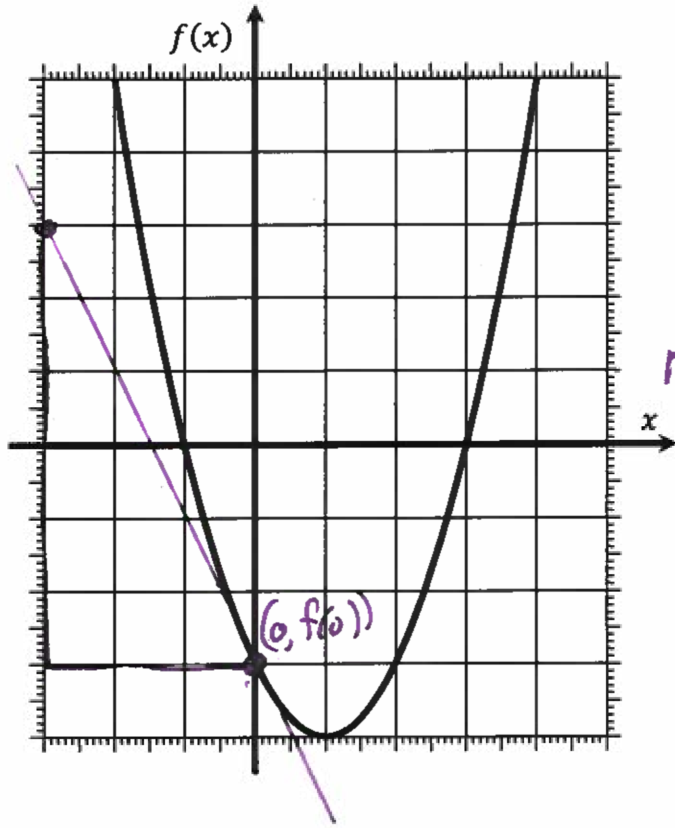
$m = \frac{\Delta y}{\Delta x} \approx \frac{-6}{1} = -6$

at $x=-1$



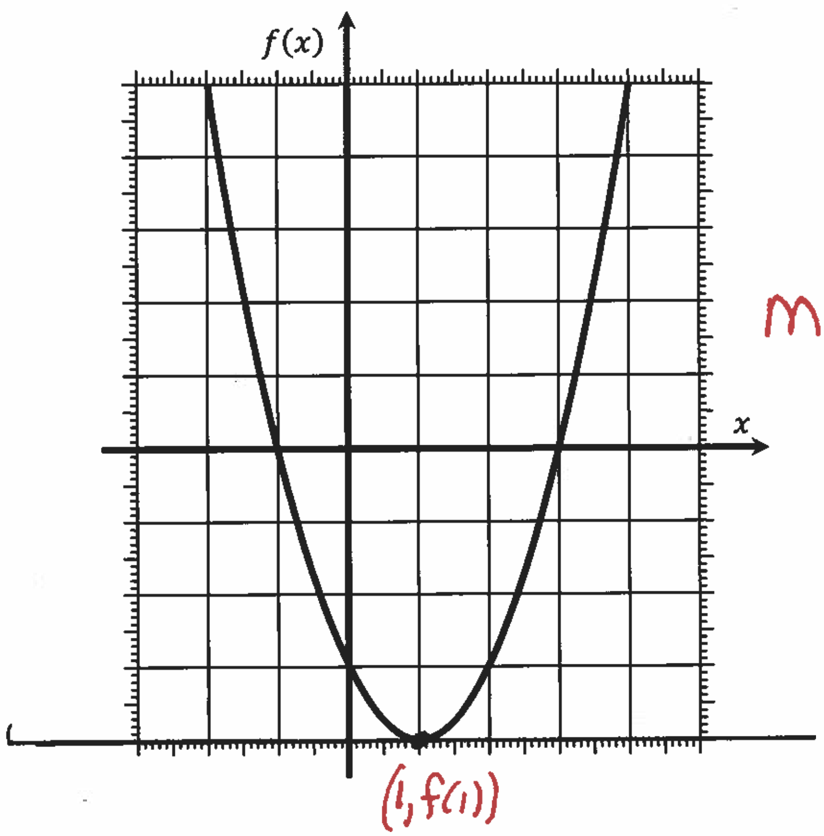
$m = \frac{\Delta y}{\Delta x} \approx \frac{-4}{1} = -4$

at $x=0$



$m = \frac{\Delta y}{\Delta x} \approx \frac{-6}{3} = -2$

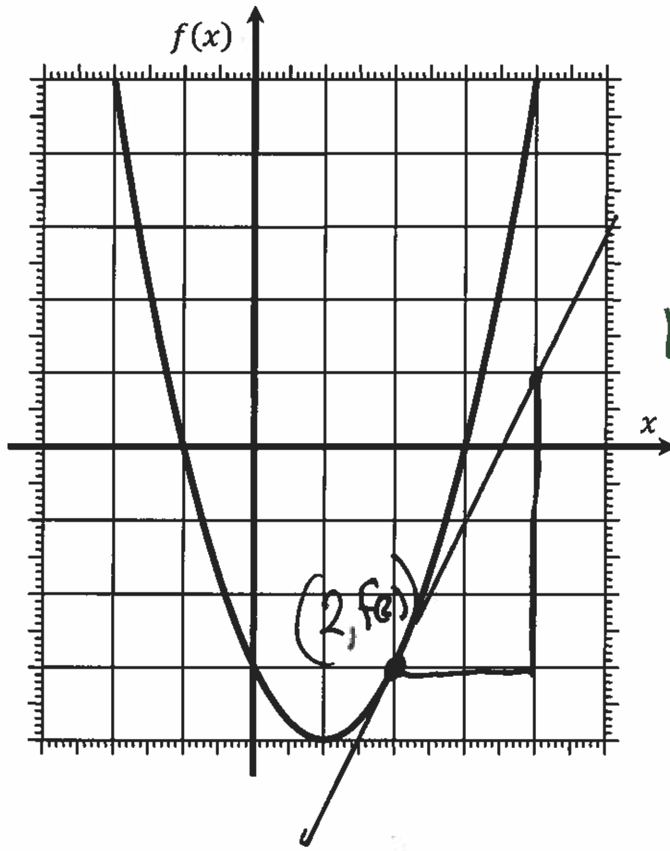
at $x=1$



$m = \frac{\Delta y}{\Delta x} = 0$

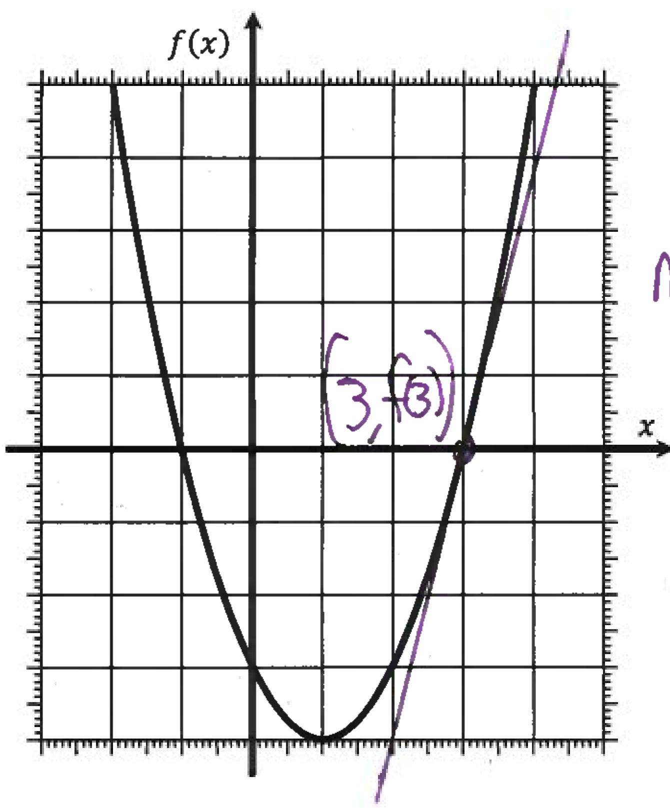
horizontal line

at $x=2$



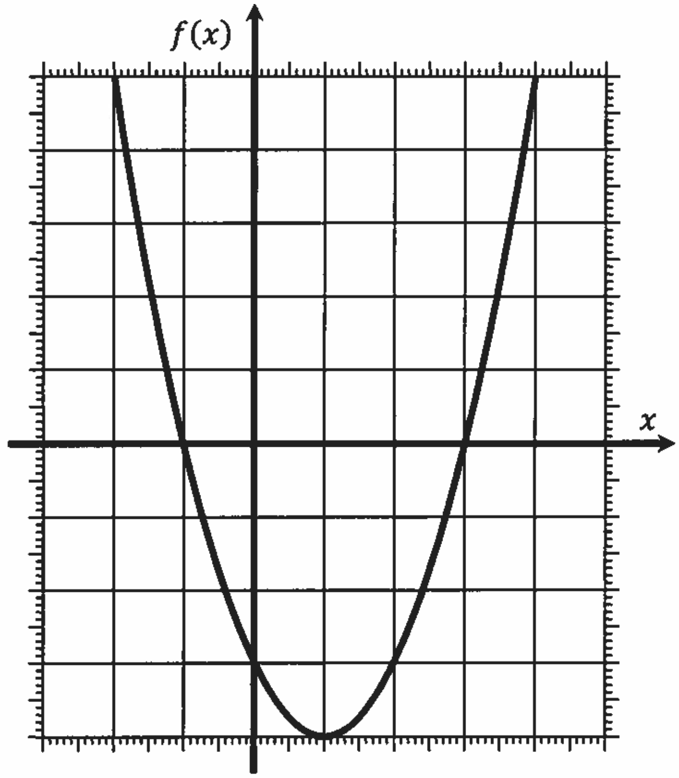
$m = \frac{\Delta y}{\Delta x} \approx \frac{4}{2} = 2$

at $x=3$

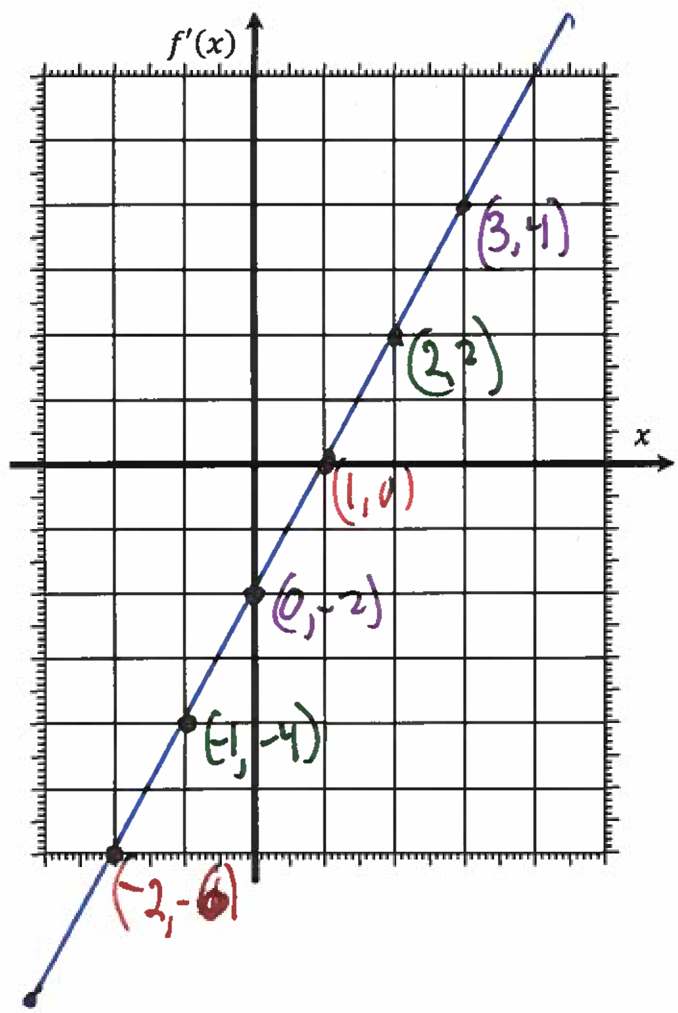


$m = \frac{\Delta y}{\Delta x} \approx \frac{4}{1} = 4$

⑧



Part 2: Using the $(x, f'(x))$ data from your table, make a graph of f'.



[Example #2] Finding the derivative of a function given by a formula.

Let $f(x) = x^2 - 2x - 3$

(a) Find $f'(x)$ using the Definition of the Derivative

Solution We have to find this limit

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Get Parts

$$f(x) = x^2 - 2x - 3$$

$$f(\ \ ) = (\ \ )^2 - 2(\ \ ) - 3 \quad \text{empty version}$$

$$f(x+h) = (x+h)^2 - 2(x+h) - 3$$

$$= x^2 + 2xh + h^2 - 2x - 2h - 3$$

Build the limit and evaluate it

$$f'(x) = \lim_{h\to 0} \frac{(x^2 + 2xh + h^2 - 2x - 2h - 3) - (x^2 - 2x - 3)}{h}$$

$$= \lim_{h\to 0} \frac{2xh + h^2 - 2h}{h}$$

indeterminate form

$$= \lim_{h\to 0} \frac{h(2x + h - 2)}{h}$$

indeterminate

Since $h \to 0$, we know $h \neq 0$ so we can cancel $\frac{h}{h}$

$$= \lim_{h\to 0} \underbrace{2x + h - 2}_{\text{polynomial in variable } h}$$

Can use direct substitution

$$= 2x + (0) - 2$$

$$= 2x - 2$$

Conclusion of question (a)

for $f(x) = x^2 - 2x - 3$

we found $f'(x) = 2x - 2$

(b) Illustrate result of (a) by graphing $f(x)$ and $f'(x)$

Solution

$f(x) = x^2 - 2x - 3 \quad = \quad (x+1)(x-3)$

Standard form — factored form

Parabola facing up

y intercept at $(0,-3)$

x intercepts at $(-1,0)$ and $(3,0)$

$f'(x) = 2x - 2$

line with slope $m = 2$

y intercept $(0,-2)$

The graphs of these would be the graphs from [Example 1]!!

[Example #3] Let $f(x) = \sqrt{x}$

(a) Find $f'(x)$ using the Definition of the derivative

Solution we need to find this limit

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

See Example #3 on page 86 of the book

Result $f'(x) = \dfrac{1}{2\sqrt{x}}$

(b) Find the slope of the line tangent to graph of $f(x) = \sqrt{x}$ at $x = 9$

Solution we need to find

$$m = f'(9) = \frac{1}{2\sqrt{9}} = \frac{1}{2 \cdot 3} = \frac{1}{6}$$

Sub $x=9$ into formula for $f'(x)$

This matches our result from Friday Sep 15 Example #1 !

[Example #4] for $G(t) = \dfrac{3t+5}{t+7}$

find $G'(t)$ using the Definition of the Derivative

Solution: See very similar Example 4 on page 86 of textbook

[Example 5] let $f(x) = \frac{5}{\sqrt{x}}$

find $f'(x)$ using definition of derivative

Solution

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{5}{\sqrt{x+h}} - \frac{5}{\sqrt{x}}}{h}$$

$$= \lim_{h \to 0} \frac{1}{h}\left(\frac{5}{\sqrt{x+h}} - \frac{5}{\sqrt{x}}\right)$$

$$= \lim_{h \to 0} \frac{5}{h}\left(\frac{1}{\sqrt{x+h}} - \frac{1}{\sqrt{x}}\right)$$

get common denom

$$= \lim_{h \to 0} \frac{5}{h}\left(\frac{1}{\sqrt{x+h}} \cdot \frac{\sqrt{x}}{\sqrt{x}} - \frac{1}{\sqrt{x}} \cdot \frac{\sqrt{x+h}}{\sqrt{x+h}}\right)$$

$$= \lim_{h\to 0} \frac{5}{h}\left(\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\cdot\sqrt{x}}\right)$$

trick: multiply and divide by special term

$$= \lim_{h\to 0} \frac{5}{h}\left(\frac{\sqrt{x} - \sqrt{x+h}}{\sqrt{x+h}\cdot\sqrt{x}}\right)\left(\frac{\sqrt{x} + \sqrt{x+h}}{\sqrt{x} + \sqrt{x+h}}\right)$$

trick Rationalize!

$$= \lim_{h\to 0} \frac{5\left(x - (x+h)\right)}{h\left(\sqrt{x+h}\cdot\sqrt{x}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

$$= \lim_{h\to 0} \frac{5(-h)}{h\left(\sqrt{x+h}\cdot\sqrt{x}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

Still indeterminate

Since $h \to 0$, we know $h \neq 0$, so we can cancel $\frac{h}{h}$

$$= \lim_{h\to 0} \frac{-5}{\left(\sqrt{x+h}\cdot\sqrt{x}\right)\left(\sqrt{x} + \sqrt{x+h}\right)}$$

no longer indeterminate

Since the limit is no longer indeterminate, we can substitute in $h=0$

$$f'(x) = -\frac{5}{\left(\sqrt{x+(0)}\cdot\sqrt{x}\right)\cdot\left(\sqrt{x}+\sqrt{x+(0)}\right)}$$

$$= -\frac{5}{\sqrt{x}\cdot\sqrt{x}\cdot\left(\sqrt{x}+\sqrt{x}\right)}$$

$$= -\frac{5}{x\cdot(2\sqrt{x})}$$

$$= \frac{-5}{2x^{3/2}}$$

Summary: for $f(x) = \dfrac{5}{\sqrt{x}}$

we find $f'(x) = \dfrac{-5}{2x^{3/2}}$

End of Lecture