MATH 2301 (Barsamian) Lecture #11 (Mon Sep 25)

Sign In

Graded work from old Quizzes can be picked up from the boxes in front of my office, Morton 521

Make Use of Isaac's + Kenny's Office Hours

Recitation R05 problems for tomorrow (Tue Sep 26) have been posted on the Course Web Page.

Quiz Q3 this Friday Sep 29, Will Cover Section 2.3

Today: Start Section 2.3 Basic Differentiation Properties

The Constant Function Rule

Two Equation Version: If $f(x) = c$, then $f'(x) = 0$

constant

Single Equation Version: $\frac{d}{dx} c = 0$

$\frac{d}{dx} c$ $\quad$ $\frac{dc}{dx}$

The Power Rule (used for finding derivative of a Power Function)

Two Equation Version If $f(x) = x^n$, then $f'(x) = n \cdot x^{n-1}$

constant exponent

variable base

Single Equation Version

$$\frac{d}{dx} x^n = n x^{n-1}$$

[Extended Example 1]

(a) $\frac{d}{dx} x^5 = 5x^{5-1} = 5x^4$

$n=5$

power rule with $n=5$

(b) $\frac{d}{dx} \frac{1}{x^5} = \frac{d}{dx} x^{-5} = (-5)x^{-5-1} = -5 \cdot x^{-6} = -5 \cdot \frac{1}{x^6} = \frac{-5}{x^6}$

$n=-5$

rewrite as a power function

power rule with $n=-5$

simplify

convert to positive exponent form

Simplify more

observations:

$$\frac{d}{dx} \frac{1}{x^5} \neq \frac{1}{\frac{d}{dx} x^5}$$

$$\frac{-5}{x^6} \neq \frac{1}{5x^4}$$

Be careful of invalid notation

$$\frac{d}{dx}\frac{1}{x^5} = x^{-5} = -5x^{-6} = -\frac{5}{x^6}$$

not true! not true!

this does not belong on the left side of the power rule

$$\frac{d}{dx}\frac{1}{x^{-5}} = \frac{d}{dx}x^{-5-1} = -5x^{-6} = \frac{-5}{x^6}$$

not true not true!

[Example 1]

(c) $\frac{d}{dx}\sqrt{x} = \frac{d}{dx}x^{1/2} = \frac{1}{2}\cdot x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2}\frac{1}{x^{1/2}} = \frac{1}{2\sqrt{x}}$

$\leftarrow n = \frac{1}{2}$

convert to power function form

power rule with $n = \frac{1}{2}$

simplify

convert to positive exponent form

much simpler than using the <u>Definition of the Derivative</u>

(see book p. 86 Example #3)

The Sum and Constant Multiple Rule

$$\frac{d}{dx}\left(a f(x) + b g(x)\right) = a \frac{d}{dx} f(x) + b \frac{d}{dx} g(x)$$

a, b constants

Example 1

(d) $\frac{d}{dx} \frac{5}{\sqrt{x}} = \frac{d}{dx} 5 \cdot x^{-1/2} = 5 \frac{d}{dx} x^{-1/2} = 5 \cdot \left(-\frac{1}{2}\right) x^{-\frac{1}{2}-1} = -\frac{5}{2} x^{-3/2} = -\frac{5}{2x^{3/2}}$

$n = -1/2$

convert to power function form

This is way simpler than using Definition of Derivative (compare to example done in Lecture Notes Mon Sep 18)

(e) $\frac{d}{dx} x^2-2x-3 = \frac{d}{dx}x^2 - 2\frac{d}{dx}x - \frac{d}{dx}3$

n=2 n=1 constant function

7

$$= \left(2x^{2-1}\right) - 2\left(1\cdot x^{1-1}\right) - \left(0\right)$$

$$= 2x^1 - 2x^0 - 0$$

$$= 2x - 2$$

Same result we got using Def. of Deriv.
on Mon Sep 18

(f) $\frac{d}{dx} x^{\pi} = \pi x^{\pi - 1}$ n = π

(g) $\frac{d}{dx} e^{\pi} = 0$ constant function!

not a power function!

(h) $\frac{d}{dx} e^{x} = ??$

we haven't learned this yet

variable exponent

constant base

exponential function

Derivatives of Sine and Cosine

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

[Example 2] Let $f(x) = 4\cos(x)$

(a) Find equation of the line tangent to graph of $f(x)$ at $x = \frac{\pi}{6}$

Solution We need to build

$$y - f(a) = f'(a)(x-a)$$

Get Parts

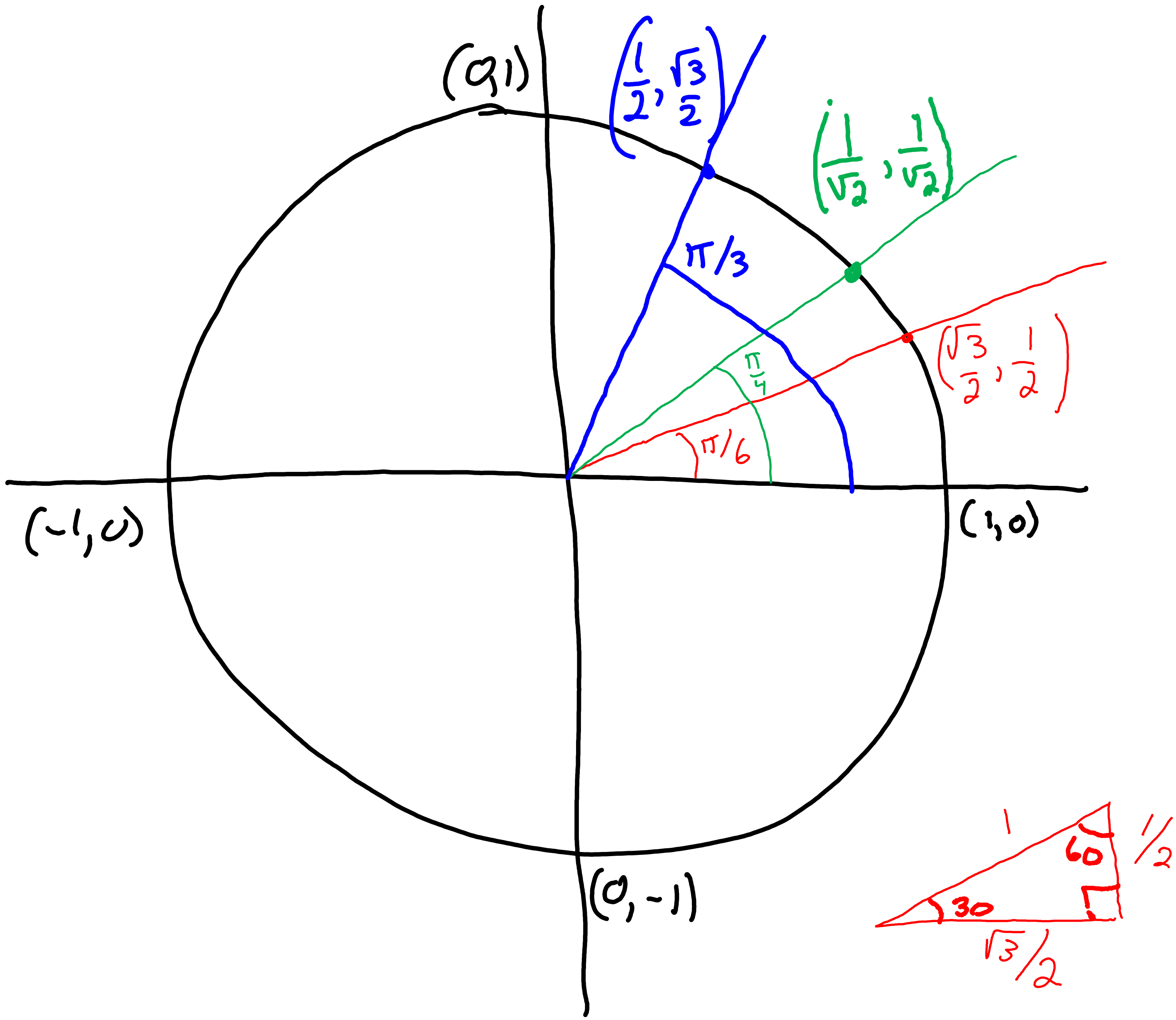
$a = \frac{\pi}{6}$ x coord of point of tangency

$f(a) = f(\pi/6) = 4\cos\left(\frac{\pi}{6}\right) = 4\left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$ y coord of point of tangency

$f'(x) = \frac{d}{dx} 4\cos(x) = 4\frac{d}{dx}\cos(x) = 4(-\sin(x)) = -4\sin(x)$

$f'(a) = f'(\pi/6) = -4\sin\left(\frac{\pi}{6}\right) = -4\left(\frac{1}{2}\right) = -2$ slope of tangent line

Review the Unit Circle in order to figure out values.



Substitute parts into equation

$$y - 2\sqrt{3} = -2\left(x - \frac{\pi}{6}\right)$$ point slope form

convert to <u>slope intercept form</u>

$$y - 2\sqrt{3} = -2x - 2\left(-\frac{\pi}{6}\right) = -2x + \frac{\pi}{3}$$

$$y = -2x + \frac{\pi}{3} + 2\sqrt{3}$$

Tangent Line Equation in Slope Intercept Form

(b) Find equation of the normal line at $x = \pi/6$

Solution

The normal line has 2 properties

- contains point of tangency
- Perpendicular to tangent line

So normal line contains $(x, y) = \left(\frac{\pi}{6}, 2\sqrt{3}\right)$

And is perp to tangent line with slope

$$m_T = -2$$

So Normal line must have slope m_N

Satisfying $m_N \cdot m_T = -1$

$$m_N = \frac{-1}{m_T} = \frac{-1}{(-2)} = \frac{1}{2}$$

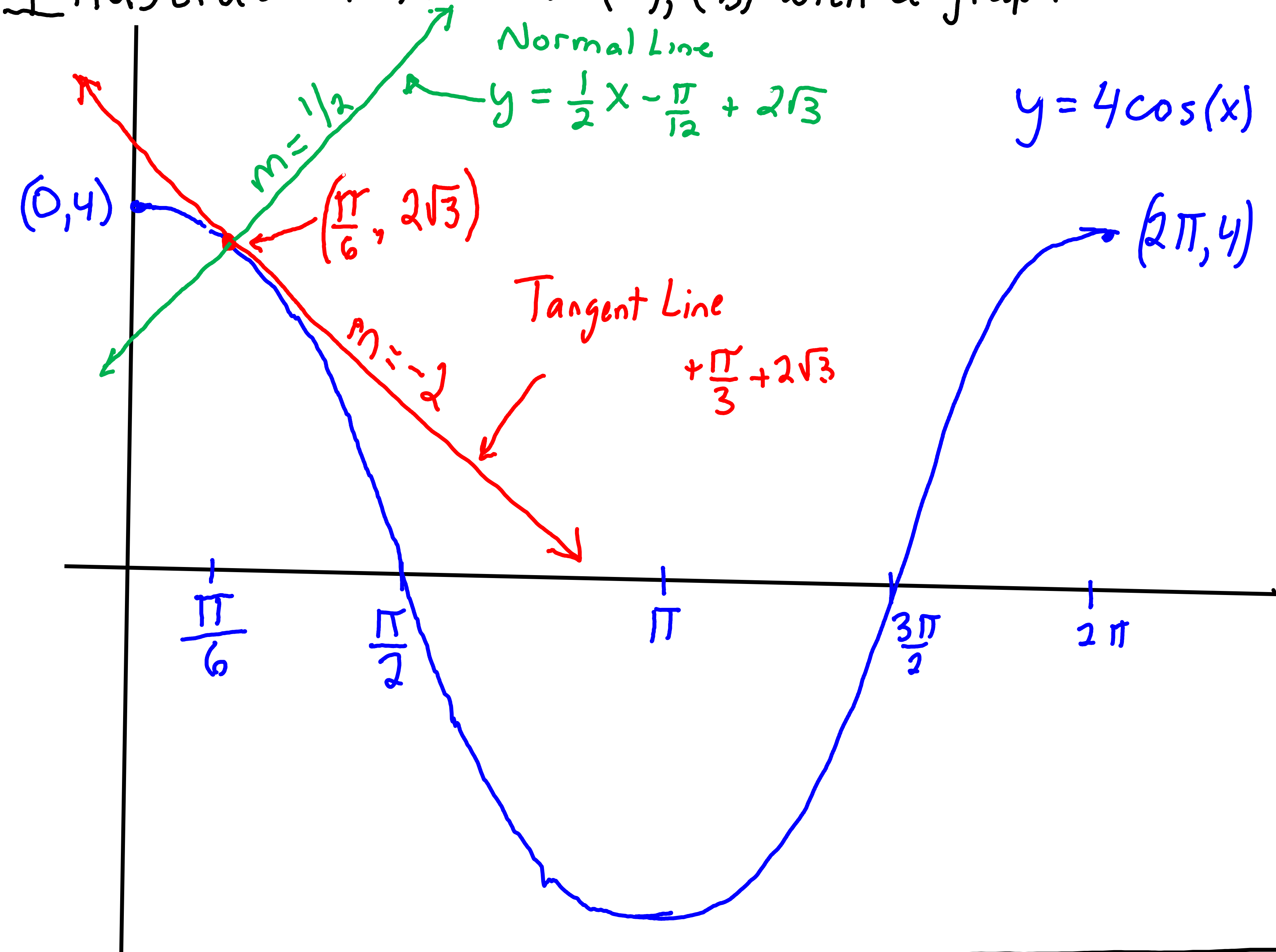
So normal line equation is

$$(y - 2\sqrt{3}) = \frac{1}{2}\left(x - \frac{\pi}{6}\right)$$

$$y = \frac{1}{2}x - \frac{\pi}{12} + 2\sqrt{3}$$

Normal Line

(c) Illustrate results of (a), (b) with a graph



End of Lecture