MATH 2301 (Barsamian) Lecture #12, Wed Sep 27, 2023

Sit in Groups of 2 or 3

Start Working on Class Drill

(I will only hand out a Class Drill to a <u>group</u>.)

Sign In

Quiz Q3 on Friday

<u>Meeting Part 1</u> Class Drill

<u>Rewriting</u> the function <u>before</u> <u>differentiating</u>

Extremely Important Skill!!

Class Drill CD05: Rewriting $f(x)$ in *Power Function Form*, then Differentiating (Section 2.5)

Part 1: Rewriting Functions in Different Forms

Fill in the empty spaces in this table.

Simplified form	=	Separate constants	=	Power Function Form. That is, a sum of terms of form *constant* × *power function* That is, $ax^p + bx^q$
$f(x) = \frac{5}{x^2} + \frac{9}{x}$	=	$5\left(\frac{1}{x^2}\right) + 9\left(\frac{1}{x}\right)$	=	$5x^{-2} + 9x^{-1}$
$f(x) = \frac{1.2}{\sqrt{x}} - \frac{0.6}{\sqrt[3]{x^2}}$	=	$1.2\left(\frac{1}{\sqrt{x}}\right) - 0.6\left(\frac{1}{\sqrt[3]{x^2}}\right)$	=	$1.2x^{-1/2} - 0.6x^{-2/3}$
$f(x) = \frac{5}{\sqrt[3]{x}} - \frac{6}{x^{1/2}}$	=	$5\left(\frac{1}{x^{1/3}}\right) - 6\left(\frac{1}{x^{1/2}}\right)$	=	$5x^{-1/3} - 6x^{-1/2}$
$-\frac{10}{x^3} - \frac{9}{x^2}$	=	$-10\left(\frac{1}{x^3}\right) - 9\left(\frac{1}{x^2}\right)$	=	$-10x^{-3} - 9x^{-2}$
$f(x) = \frac{7\sqrt[3]{x}}{5} + \frac{3}{11x^{2/5}}$	=	$\frac{7}{5} \cdot x^{1/3} + \frac{3}{11} \cdot \frac{1}{x^{2/5}}$	=	$\frac{7}{5}x^{1/3} + \frac{3}{11} \cdot x^{-2/5}$
$\frac{7}{15x^{2/3}} - \frac{6}{55x^{7/5}}$	=	$\frac{7}{15}\left(\frac{1}{x^{2/3}}\right) - \frac{6}{55}\left(\frac{1}{x^{7/5}}\right)$	=	$\frac{7}{15}x^{-2/3} - \frac{6}{55}x^{-7/5}$

Part 2 is on back ➔

Part 2: Finding a Derivative Using Sum Rule, Constant Multiple Rule, Power Rule

$$f(x) = \frac{7\sqrt[3]{x}}{5} + \frac{3}{11x^{2/5}}$$

(A) Rewrite $f(x)$ in ***power function form***.
That is, rewrite it as a sum of terms of the form *constant* × *power function*. That is, $ax^p + bx^q$.
(**Hint:** You have already done this part on the previous page!)

$$f(x) = \frac{7}{5}x^{1/3} + \frac{3}{11}x^{-2/5}$$

(B) Find $f'(x)$.

- Use the techniques of Section 2.5. (That is, DO NOT use the Definition of the Derivative.)
- Show all details clearly and use correct notation.
- Simplify your final answer, and rewrite it so that it does not have any negative exponents. (**Hint:** You have already done the necessary simplifying on the previous page!)

$$f'(x) = \frac{d}{dx}\left(\frac{7}{5}x^{1/3} + \frac{3}{11}x^{-2/5}\right)$$

$$= \frac{7}{5}\frac{d}{dx}x^{1/3} + \frac{3}{11}\frac{d}{dx}x^{-2/5}$$

$n = 1/3$ $\quad$ $n = -\frac{2}{5}$

$$= \frac{7}{5}\left(\frac{1}{3}x^{1/3-1}\right) + \frac{3}{11}\left(\left(-\frac{2}{5}\right)x^{-\frac{2}{5}-1}\right)$$

$$= \frac{7}{15}x^{-\frac{2}{3}} - \frac{6}{55}x^{-7/5}$$

$$\vdots$$

$$= \frac{7}{15x^{2/3}} - \frac{6}{55x^{7/5}}$$

Meeting Part 2: Trick Problems

[Example 1] find derivative of $f(x) = \dfrac{x^3 - 12x + 7}{\sqrt{x}}$

Solution Rewrite $f(x)$ in power function form first, then differentiate

Rewrite

$$f(x) = \frac{x^3}{\sqrt{x}} - \frac{12x}{\sqrt{x}} + \frac{7}{\sqrt{x}} = x^{5/2} - 12x^{1/2} + 7x^{-1/2}$$

Differentiate

$$f'(x) = \frac{d}{dx} x^{5/2} - 12\frac{d}{dx} x^{1/2} + 7\frac{d}{dx} x^{-1/2}$$

$$= \frac{5}{2}x^{5/2-1} - 12\left(\frac{1}{2}x^{1/2-1}\right) + 7\left(-\frac{1}{2}\right)x^{-1/2-1}$$

$$= \frac{5}{2}x^{3/2} - 6x^{-1/2} - \frac{7}{2}x^{-3/2}$$

$$= \frac{5}{2}x^{3/2} - \frac{6}{\sqrt{x}} - \frac{7}{2x^{3/2}}$$

[Example 2] Find the value of $\lim_{h\to 0} \frac{(5+h)^4 - 625}{h}$

Solution: Notice: there are no instructions given about <u>how</u> to get answer.
Consider this diagram

<u>given $f(x) = x^4$ and $a=5$, find $f'(a)$</u>

using Section 2.1 techniques (build the limit)

$\lim_{h\to 0} \frac{(5+h)^4 - 625}{h}$

using Section 2.3 techniques (derivative rules)

$f'(x) = 4x^3$

This green path is <u>easy</u>

evaluate the limit (<u>really</u> <u>hard</u>)

Substitute $x=5$

$f'(5) = 4(5)^3 = 4 \cdot 125$

500

Example 3 Find the value of

$$\lim_{h \to 0} \frac{\cos\left(\frac{\pi}{3}+h\right) - \frac{1}{2}}{h}$$

Trick: Observe that this is asking for $f'(a)$

where $f(x) = \cos(x)$

$a = \pi/3$

check: the form $f'(\pi/3) = \lim_{h \to 0} \frac{\cos(\pi/3 + h) - \cos(\pi/3)}{h}$

this form does match what we were given.

get $f'(x) = \frac{d}{dx} \cos(x) = -\sin(x)$

$f'(a) = f'(\pi/3) = -\sin(\pi/3) = -\frac{\sqrt{3}}{2}$ answer

Meeting Part 3 Position, Velocity, Acceleration

An object moves in 1 dimension with position function

$$s(t) = f(t) = t^3 - 12t^2 + 36t$$

position

f is not helpful

t = time in seconds

$s(t)$ = position (at time t) in feet.

(a) Find velocity at time t.

Solution

velocity $v(t) = s'(t) = \frac{d}{dt}\left(t^3 - 12t^2 + 36t\right) = 3t^2 - 24t + 36$

(b) Find velocity at time 3 seconds

Solution

$$v(3) = 3(3)^2 - 24(3) + 36 = 27 - 72 + 36 = -9 \ \frac{ft}{sec}$$

Sub $t=3$

(c) Find times when object is at rest (not moving)

Solution: Set velocity $= v(t) = 0$, solve for t

$$0 = v(t) = 3t^2 - 24t + 36$$

$$= 3(t^2 - 8t + 12)$$

$$= 3(t-2)(t-6)$$

Solutions $t = 2$, $t = 6$

Questions that we did not get to

(d) At what times is the object moving in the <u>forward</u> direction

(e) Find the total distance travelled by the object in the first 8 seconds.

(f) Draw a diagram to illustrate the movement of the object during the first 8 seconds

(g) Find the acceleration $a(t)$ at time t and at time $t = 3$ seconds.

(h) Graph $s(t)$, $v(t)$, $a(t)$ together on a single set of axes.

→ This <u>whole example</u>, including problems (d)-(h) that we did not get to in class, is <u>very similar</u> to Textbook Section 2.3 Example 9 on pages 102-103. Study that example, then answer questions (d)-(h) above.

End of lecture