# MATH 2301 (Barsamian) Lecture #13, Fri Sep 29, 2023

## Pick Up Graded Exams

- Remember that all Quiz & Exam Problems are based on exercises from the Homework list.
- Remember that I don't cover all the exercises in Lecture. Exercises that have excellent similar examples in the book, I will try to not cover in class. ([7], [8])
- A distressing number of you did this:

$$\sqrt{a+b} = \sqrt{a} + \sqrt{b}$$

## Quiz Q3 Today

# Section 2.4 The Product Rule and Quotient Rule



**The Product Rule** (for finding derivative of a Product of functions)

$$\frac{d}{dx}\left(f(x)g(x)\right) = \left(\frac{d}{dx}f(x)\right)\cdot g(x) + f(x)\left(\frac{d}{dx}g(x)\right)$$

Observe: $\frac{d}{dx}\left(f(x)g(x)\right) \neq \left(\frac{d}{dx}f(x)\right)\left(\frac{d}{dx}g(x)\right)$

[Example 1] $\frac{d}{dx}\left(\sqrt{x}\sin(x)\right) = \left(\frac{d}{dx}\sqrt{x}\right)\sin(x) + \sqrt{x}\left(\frac{d}{dx}\sin(x)\right)$

↑ product rule

$$= \left(\frac{1}{2\sqrt{x}}\right)\sin(x) + \sqrt{x}\cdot\cos(x)$$

Be careful of handwriting!

Don't write $\frac{1}{2\sqrt{x}}\sin(x)$ or $1/2\sqrt{x}\sin(x)$

**The Quotient Rule** (for finding Derivative of a Quotient)

$$\frac{d}{dx}\left(\frac{\text{top}(x)}{\text{bottom}(x)}\right) = \frac{\left(\frac{d}{dx}\text{top}(x)\right)\text{bottom}(x) - \text{top}(x)\left(\frac{d}{dx}\text{bottom}(x)\right)}{(\text{bottom}(x))^2}$$

[Example] $\frac{d}{dt}\left(\frac{5t}{5+\sqrt{t}}\right) = \frac{\left(\frac{d}{dt}5t\right)(5+\sqrt{t}) - 5t\left(\frac{d}{dt}5+\sqrt{t}\right)}{(5+\sqrt{t})^2}$

note: cannot cancel $5+\sqrt{t}$ here!

$$= \frac{(5)(5+\sqrt{t}) - 5t\left(\frac{1}{2\sqrt{t}}\right)}{(5+\sqrt{t})^2}$$

$$= \frac{25 + 5\sqrt{t} - \frac{5}{2}\sqrt{t}}{(5+\sqrt{t})^2}$$

$$= \frac{25 + \frac{5}{2}\sqrt{t}}{(5+\sqrt{t})^2}$$

[Example] Find derivative of $f(x) = \dfrac{x^3 - 12x + 7}{\sqrt{x}}$

Solution

$$\frac{d}{dx}\left(\frac{x^3-12x+7}{\sqrt{x}}\right) = \frac{\left(\frac{d}{dx}x^3-12x+7\right)\cdot\sqrt{x} - \left(x^3-12x+7\right)\frac{d}{dx}\sqrt{x}}{(\sqrt{x})^2}$$

really hard ...

Would lead to correct answer, but would be dangerous.

Smart way to do this derivative (done in class on Wed Sep 27, on page 4)

rewrite $f(x)$ first. $f(x) = x^{5/2} - 12x^{1/2} + 7x^{-1/2}$

Then find derivative using much easier rules

$$f'(x) = \frac{d}{dx}\left(x^{5/2} - 12x^{1/2} + 7x^{-1/2}\right) = \cdots = \frac{5}{2}x^{3/2} - \frac{6}{\sqrt{x}} - \frac{7}{2x^{3/2}}$$

↑ easier steps

# Derivative of tan(x)

$$\frac{d}{dx}\tan(x) = \left(\frac{d}{dx}\frac{\sin(x)}{\cos(x)}\right) = \cdots = (\sec(x))^2$$

$\cdots$ ↑ Quotient Rule

See details of calculation in book Section 2.4, Page 111

## Derivatives of Trig Functions

$$\frac{d}{dx}\sin(x) = \cos(x)$$

$$\frac{d}{dx}\cos(x) = -\sin(x)$$

$$\frac{d}{dx}\tan(x) = (\sec(x))^2$$

$$\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)$$

$$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$$

$$\frac{d}{dx}\cot(x) = -(\csc(x))^2$$

end of Lecture