MATH 2301 (Barsamian) Lecture #14 (Mon Oct 2, 2023)

Sign In

Recitation Assignments for tomorrow (Tue Oct 3) are posted

Quiz Q4 this coming Friday

Be sure to read the textbook

(either online eText in WebAssign or paper book from bookstore)

Examples that are done well in the book

- I tend to put similar exercises on the Homework List
- I tend to <u>not</u> do those examples in lecture.

Do as many of the Recitation problems as you can before Recitation. (They are <u>all</u> HW problems!)

Get a <u>free binder</u>! Organize your work!

Section 2.5 The Chain Rule

The chain rule is used for taking the derivative of nested function, Functions of the form

$$\text{outer}(\text{inner}(x))$$

Common Mathematical Terminolog and Notation

Composition of functions

$$\text{Outer} \circ \text{inner}(x) \quad \text{or} \quad f \circ g(x)$$

This common notation is not helpful for use.

The Chain Rule

$$\frac{d}{dx}\,\text{outer}(\text{inner}(x)) = \text{Outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

Note: This is not the notation used by our book or almost any book.

Three Examples where Outer Function is a Power Function

[Example 1] Let $f(x) = (x^3-12x+17)^5$. Find $f'(x)$.

$$f'(x) = \frac{d}{dx}(x^3-12x+17)^5$$

$$= \frac{d}{dx}\,\text{outer}(\text{inner}(x))$$

Chain Rule

$$= \text{outer}'(\text{inner}(x))\cdot\text{inner}'(x)$$

$$= 5(x^3-12x+17)^4\cdot(3x^2-12)$$

very important parentheses!

Chain Rule Details

$\text{inner}(x) = x^3-12x+17$

$\text{inner}'(x) = 3x^2-12$

$\text{outer}(\quad) = (\quad)^5$ power function

empty version

$\text{outer}'(\quad) = 5(\quad)^4$

[Example 2] Let $f(x) = \dfrac{1}{(x^3-12x+17)^5}$ Find $f'(x)$

Looks like this derivative would involve both Quotient Rule & Chain Rule.
This would work but would be many steps & dangerous!

Better Approach: Rewrite function $f(x) = (x^3-12x+17)^{-5}$

$$f'(x) = \frac{d}{dx}(x^3-12x+17)^{-5}$$

$$= \frac{d}{dx}\,\text{outer}(\text{inner}(x))$$

chain rule

$$= \text{outer}'(\text{inner}(x)) \cdot \text{inner}'(x)$$

Chain Rule Details

$\text{inner}(x) = x^3-12x+17$

$\text{inner}'(x) = 3x^2-12$

$\text{outer}(\ \ ) = (\ \ \ )^{-5}$ power function

empty version

$\text{outer}'(\ \ ) = -5(\ \ \ )^{-5-1} = \underbrace{-5(\ \ )^{-6}}$ $\cancel{\overset{?}{=} -5(\ \ )^{6}}$

$= -5\dfrac{1}{(\ \ )^6} = -\dfrac{5}{(\ \ \ )^6}$

$$= -\frac{5}{(x^3-12x+17)^6} \cdot (3x^2-12)$$

$$= -\frac{5(3x^2-12)}{(x^3-12x+17)^6}$$

[Example 3] Let $f(x) = \sqrt{x^3 - 12x + 17}$

ⓐ find $f'(x)$

Solution

$$f'(x) = \frac{d}{dx}\left(x^3 - 12x + 17\right)^{1/2}$$

$$= \frac{d}{dx}\, outer(inner(x))$$

chain rule

$$= outer'(inner(x)) \cdot inner'(x)$$

$$= \frac{1}{2\sqrt{(x^3 - 12x + 17)}} \cdot (3x^2 - 12)$$

$$= \boxed{\frac{3x^2 - 12}{2\sqrt{x^3 - 12x + 17}} = f'(x)}$$

Chain Rule Details

$inner(x) = x^3 - 12x + 17$

$inner'(x) = 3x^2 - 12$

$outer(\ \ ) = (\ \ \ )^{1/2}$ power function

empty version

$outer'(\ \ ) = \frac{1}{2}(\ \ \ )^{\frac{1}{2}-1} = \frac{1}{2}(\ \ \ )^{-\frac{1}{2}}$

$= \frac{1}{2}\frac{1}{(\ \ \ )^{1/2}} = \frac{1}{2(\ \ )^{1/2}} = \frac{1}{2\sqrt{(\ \ )}}$

(b) Find the slope of the line that is tangent to graph of f at $x=0$

Solution $m = f'(0) = \dfrac{3(0)^2-12}{2\sqrt{(0)^3-12(0)+17}} = \dfrac{-12}{2\sqrt{17}}$ (roughly $-\frac{3}{2}$)

Sub $x=0$ into $f'(x)$

(c) Find x coordinates of all points on graph of f that have horizontal tangent lines.

Solution Set $f'(x)=0$ and solve for x because horiz lines have slope $m=0$

$$m = 0 = f'(x) = \frac{3x^2-12}{2\sqrt{x^3-12x+17}}$$

remember: A fraction $\frac{a}{b}=0$ only when $a=0$ and $b\neq 0$

Set numerator $= 0$

$$0 = 3x^2 - 12 = 3(x^2 - 4) = 3(x+2)(x-2)$$

Solutions: $x = -2$ or $x = 2$

notice 3 is not on this list, because if we substitute $x = 3$, we get $3((3)+2)((3)-2) = 3(5)(1) \neq 0$

So the only candidates are $x=-2$ and $x=2$.
But we must confirm that the denominator is not zero.

when $x = -2$, denominator $= 2\sqrt{(-2)^3 - 12(-2) + 17} = 2\sqrt{-8+24+17} = 2\sqrt{33} \neq 0$

when $x = 2$, denominator $= 2\sqrt{(2)^3 - 12(2) + 17} = 2\sqrt{1} = 2 \neq 0$

So the horizontal tangent lines do occur at $x=-2$ and $x=2$

[Example 4] Let $f(x) = \tan^2(\cos(x))$ find $f'(x)$

Solution

$$f'(x) = \frac{d}{dx}\left(\tan(\cos(x))\right)^2$$

Chain Rule Details

$inner(x) = \tan(\cos(x))$

$inner'(x) = \frac{d}{dx}\tan(\cos(x))$

$outer(\ \ ) = (\ \ )^2$

$outer'(\ \ ) = 2(\ \ )' = 2(\ \ )$

$$= 2\left(\tan(\cos(x))\right) \cdot \frac{d}{dx}\tan(\cos(x))$$

Chain Rule Details

$inner(x) = \cos(x)$

$inner'(x) = -\sin(x)$

$outer(\ \ ) = \tan(\ \ )$

$outer'(\ \ ) = \sec^2(\ \ )$

$$= 2\tan(\cos(x)) \cdot \sec^2(\cos(x)) \cdot (-\sin(x))$$

End of Lecture