MATH 2301 (Barsamian) Lecture #16 (Fri Oct 6, 2023) ①

Pick up a handout

Sit in alternate seats

Sign In

Quiz Q4 today

Remember

Quiz Q5 next Wednesday, Oct 11

Old graded work is in the boxes outside my office, Morton 521

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Implicit Differentiation and Related Rates

MATH 2301 (Barsamian), October 2023

The Method of Implicit Differentiation (Section 2.6)

(Used for finding y' when x, y are related by an equation that is not solved for y.)

Starting with: An equation involving x and y.

Step 1: Take derivative of left and right sides of this new equation with respect to x. Keep in mind the difference between taking the derivative of x and taking the derivative of y.

$$\frac{d}{dx}x \underset{\substack{power \\ rule \\ with\; n=1}}{=} 1$$

$$\frac{d}{dx}y = y' \quad \text{This is unknown! We cannot go any farther.}$$

The result will be a new equation involving x and y and y'.

Step 2: Solve for y'. The result will be a new equation of the form

$$y' = \text{expression involving } x \text{ and } y$$

Recall the Definition of Instantaneous of Change for a function $f(t)$

Words: *Instantaneous Rate of Change of* $f(t)$ *at time* $t = a$.

Symbol: $f'(a)$

Spoken: The derivative of f at a.

Units: The units of $f'(a)$ are $\frac{\text{units of the quantity } f(t)}{\text{units of time } t}$

Additional terminology: When the variable is t, representing time and the function $f(t)$ is a position function, representing the position of an object at time t, then the Instantaneous rate of change is called the *instantaneous velocity at time* $t = a$.

Related Rates problems: (Section 2.7)

Given various quantities that are related by an equation,
and given values for certain of the quantities or rates of change of those quantities
Find some unknown rate of change, or find some unknown quantity.

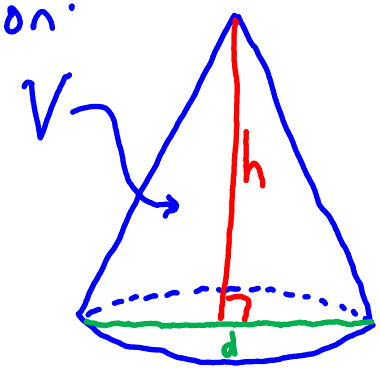
Solving Related Rates Problems

- Draw a picture of the situation
- Label the picture with variables for important quantities
- Label the picture with numbers for known quantities (or known rates)
- Identify the goal: find some unknown rate or find some unknown quantity.
- Find an equation relating the variables.
- Differentiate both sides of this equation with respect to time t using the method of ***Implicit Differentiation***. The result will be a **new equation** relating the quantities and the derivatives of the quantities. That is, this new equation describes a relationship between the rates of change of the quantities. Hence, the name ***Related Rates***.
- Solve this new equation for the unknown derivative or the unknown quantity.
- Substitute in values.

[Example 1] Involving volume of basic shapes. (Similar to 2.7 #27)

Gravel is being dumped at a rate of 50 ft^3/min, forming a conical pile whose height and base diameter are always equal. How fast is the height growing at the instant when the height is 20 ft?

Solution:



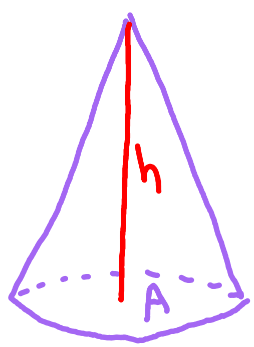
$h = 20\,ft$

$V' = 50 \frac{ft^3}{min}$

$h = d$

goal: find h'

Recall



$V = A \cdot h$

$d = h$

$V = \frac{1}{3} A h = \frac{1}{3}(\pi r^2) h = \frac{\pi}{3}\left(\frac{d}{2}\right)^2 h = \frac{\pi d^2 h}{12} = \frac{\pi h^2 h}{12} = \frac{\pi h^3}{12}$

$$V = \frac{\pi h^3}{12}$$ equation relating variable h, V

$$\frac{d}{dt}(V) = \frac{d}{dt}\left(\frac{\pi}{12} h^3\right)$$

$$V' = \frac{\pi}{12} \frac{d}{dt} \underbrace{h^3}_{\text{nested function}}$$

$(h)^3$

need chain rule

Chain Rule Details

inner(t) = h

inner'(t) = h'

outer( ) = $(\ )^3$

outer'( ) = $3(\ )^2$

$$V' = \frac{\pi}{12} \cdot 3(h)^2 \cdot h' = \frac{\pi h^2 h'}{4}$$ equation relating h, V, h', V'

Solve for h'

$$h' = \frac{4V'}{\pi h^2}$$

Substitute in known values

$$h' = \frac{4 \cdot 50\ ft^3/min}{\pi (20 ft)^2} = \frac{200\ ft^3}{\pi\ min\ 20^2\ ft^2} = \frac{200}{\pi 400} \frac{ft}{min} = \frac{1}{2\pi} \frac{ft}{min}$$

End of Lecture