MATH 2301 (Barsamian) Lecture #18, Wed Oct 11, 2023

Pick Up Graded Work

Sit in Alternate Seats (Quiz Q5 today)

Sign In

Holiday Friday

Exam X2 Next Friday Oct 20

Common Mistakes on Quiz Q4

[1] many of you did not use product rule

[2] Many of you only found $f'(x)$ or $f'(1)$. Remember that when you are asked to find equation of tangent line, start problem by writing that you are going to build $(y-f(a)) = f'(a)(x-a)$

[3] Quotient rule unnecessary, too hard, prone to errors.
Rewrite $f(x) = \dfrac{5}{\sqrt{x^2+bx+c}} \underset{\text{rewrite}}{=} 5(x^2+bx+c)^{-1/2}$. Then use chain rule with outer$(\ ) = 5(\ )^{-1/2}$

[4] The derivative of BLAH is written $\dfrac{d}{dx}(BLAH)$, not $\dfrac{dy}{dx}BLAH$

Today: More about Section 2.8: Differentials

Remark: I won't discuss 3.1, 3.2 in a full lecture because they are prerequisite topics. But you have webAssign Homework on those sections.

Differentials Consider a function f and two known x values, x_1 and x_2

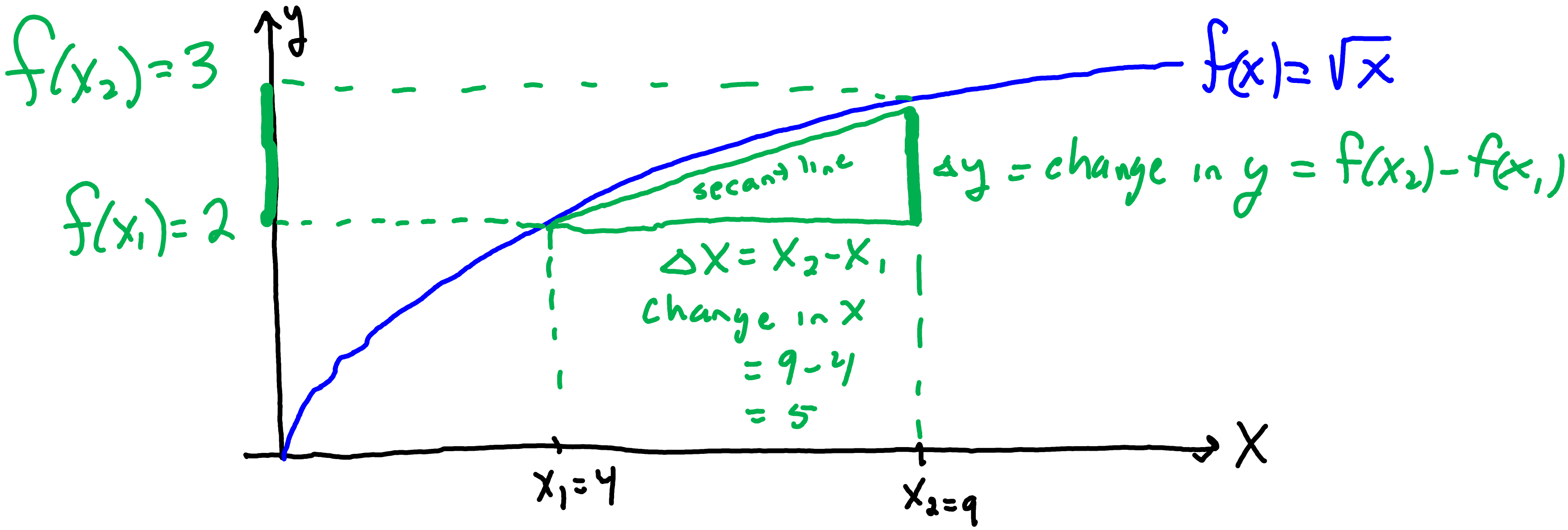
How much does value of f change when x changes from x_1 to x_2?

Change in $f(x)$ $= \Delta y = f(x_2) - f(x_1)$

delta refers to "change"

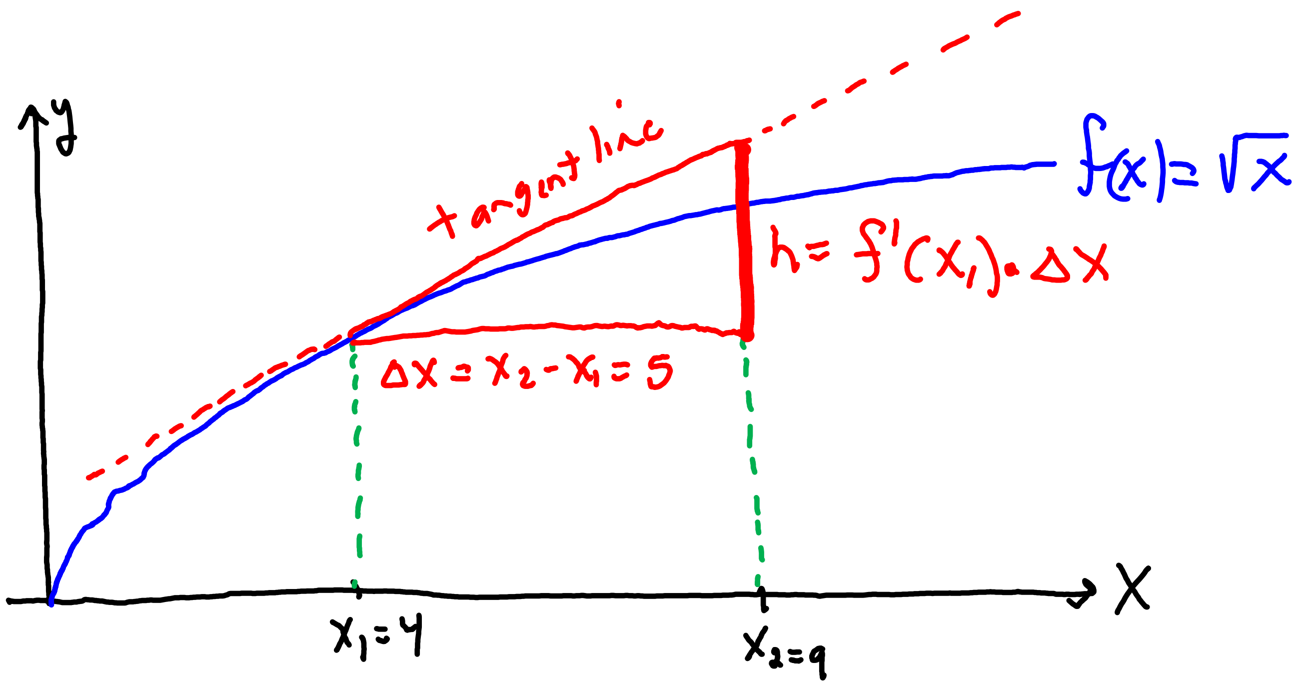
Example $f(x) = \sqrt{x}$, $x_1 = 4$ and $x_2 = 9$

then $\Delta y = f(x_2) - f(x_1) = \sqrt{9} - \sqrt{4} = 3 - 2 = 1$



$f(x_2)=3$
$f(x_1)=2$
y
$f(x)=\sqrt{x}$
secant line
Δy = change in y = $f(x_2)-f(x_1)$
$\Delta x = x_2 - x_1$
change in x
$= 9-4$
$= 5$
x
$x_1=4$
$x_2=9$

There is <u>another</u> triangle that could be drawn, involving a different height



tangent line slope $m = f'(x_1) = f'(4)$

but also, slope $m = \dfrac{\text{rise}}{\text{run}} = \dfrac{h}{\Delta x}$

so $f'(x_1) = \dfrac{h}{\Delta x}$

multiply both sides by Δx

$h = f'(x_1) \cdot \Delta x$

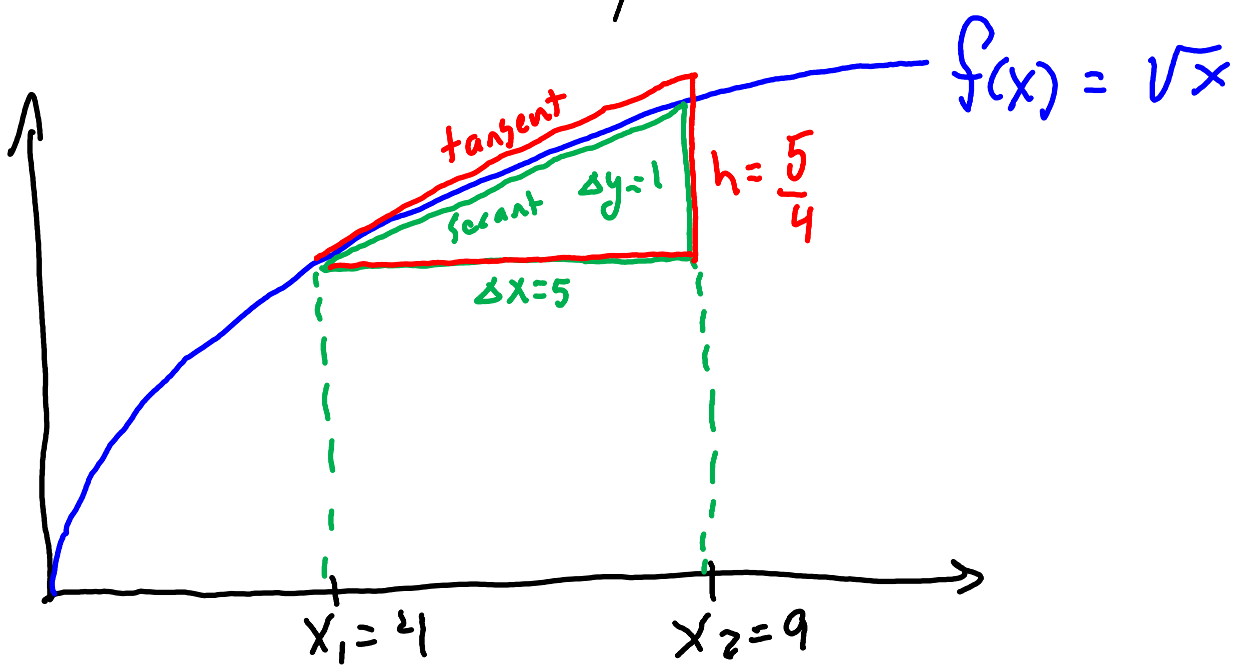
Find value for $h = f'(x_1) \cdot \Delta x = f'(4) \cdot 5$

$$f(x) = \sqrt{x}$$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

$$f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{2 \cdot 2} = \frac{1}{4}$$

$$h = \frac{1}{4} \cdot 5 = \frac{5}{4}$$



The quantity $h = f'(x_1) \cdot \Delta x$ is useful and has a name

<u>Definition</u>

Words: The <u>differential of f at x_1</u>, when $\Delta x = x_2 - x_1$

symbol: dy

meaning: $dy = f'(x_1) \cdot \Delta x$

<u>Usefulness!</u>

Often interested in Δy, but Δy can be hard to compute While dy is often <u>easier</u> to compute and is often very close to Δy, especially when Δx is <u>small</u>.

Example Recall $V_{sphere} = \frac{4}{3}\pi r^3$

$$V_{hemisphere} = \frac{2\pi r^3}{3}$$

Suppose $r_1 = 50$ and $r_2 = 50.0005$

Find Δy and dy

Solution

$$\Delta y = \Delta V_{hemisphere} = V(r_2) - V(r_1) =$$

$$= \frac{2}{3}\pi(50.0005)^3 - \frac{2}{3}\pi(50)^3$$

$$= \frac{2\pi}{3}\left(50.0005^3 - 50^3\right)$$

really hard! $\underline{\underline{\approx}}$ (calculator) 7.8540602

Now get $dy = dV = V'(r_1) \cdot \Delta r$

$$\Delta r = 50.0005 - 50 = 0.0005$$

$$V(r) = \frac{2\pi r^3}{3}$$

$$V'(r) = \frac{d}{dr}\frac{2\pi r^3}{3} = \frac{2\pi}{3}\frac{d}{dr}r^3 = \frac{2\pi}{\cancel{3}} \cdot \cancel{3}r^2 = 2\pi r^2$$

$$V'(r_1) = 2\pi(50)^2 = 2\pi \cdot 2500$$

$$dV = 2\pi \cdot 2500 \cdot (0.0005) = \pi \cdot 2500 \cdot (0.001)$$

$= 2.5\pi$ about 7 or 8

$\underset{\text{calculator}}{=}$ 7.85398 Very close to ΔV !!

Example Building has hemispherical dome with radius $r = 50\,ft$

(HW problem)

you need to paint the dome

Paint thickness needs to be $0.0005\ ft$.

What is the volume of paint that you need?

Exact & approximate.

Solution

Exact $= \Delta V = 7.8540602\ ft^3 \approx 58.75200408$ gallons

Approx $= dV = 7.85398\,ft^3 \approx 58.75126$ gallons

So you will need to buy 59 gallons of paint.

Observe that the approximation dV was much simpler to compute than ΔV, and gave a perfectly useful number!

End of lecture