MATH2301 (Barsamian) Lecture #19 (Mon Oct 16)

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Recitation Problems for Tomorrow are Posted

Exam X2 on Friday

Today: Section 3.3: Derivatives of Logarithmic + Exponential Functions

Functions Involving Exponents

$f(x) = x^n$ ← constant exponent ← variable base

Power Function

$f(x) = e^{(x)}$ ← variable exponent ← constant base

exponential function, base e

natural exponential function

$f(x) = b^{(x)}$ ← variable exponent ← constant base

exponential function

$f(x) = x^x$ ← variable exponent ← variable base

Derivative

$$\frac{d}{dx} x^n = n \cdot x^{n-1}$$

power rule

$$\frac{d}{dx} e^{(x)} = e^{(x)}$$

exponential function rule base e

$$\frac{d}{dx} b^{(x)} = b^{(x)} \cdot \ln(b)$$

general exponential function rule

???

Stay tuned

[Example 1] $f(x) = \cot(x)\cdot e^{(x)} + \cot(e^{(x)}) + e^{\cot(x)}$

Find $f'(x)$

Solution find each derivative separately

$$\frac{d}{dx}\Big(\underbrace{\cot(x)\cdot e^{(x)}}_{\text{Product}}\Big) \underset{\text{product rule}}{=} (\cot(x))'\cdot e^{(x)} + \cot(x)\cdot(e^{(x)})' = -\csc^2(x)\cdot e^{(x)} + \cot(x)\cdot e^{(x)}$$

$$= (-\csc^2(x) + \cot(x))\cdot e^{(x)}$$

$$\frac{d}{dx}\cot(e^{(x)}) = -\csc^2(e^{(x)})\cdot e^{(x)}$$

Chain Rule Details

inner(x) = $e^{(x)}$

inner'(x) = $e^{(x)}$

outer( ) = cot( )

outer'( ) = $-\csc^2($)

(4)

$$\frac{d}{dx} e^{(\cot(x))} = e^{(\cot(x))}\left(-\csc^2(x)\right)$$

need these!

Chain Rule Details

inner$(x) = \cot(x)$

inner$'(x) = -\csc^2(x)$

outer$(\ ) = e^{(\ )}$ empty function

outer$'(\ ) = e^{(\ )}$

common mistake

~~outer$(\ ) = e$~~ constant

~~outer$'(\ ) = 0$~~

outer$'$(inner(x)) $\cdot$ inner$'(x)$

So $f'(x) =$ BLAH + BLAH + BLAH

[Example 2] Let $f(x) = e^{(-x^2-2x-1)}$

(a) Find slope of the line that is tangent to graph of $f(x)$ at $x=0$

Solution: we need to find $m = f'(0)$

Strategy: • find $f'(x)$
• substitute in $x=0$ to get $f'(0)$

not this
$y - f(a) = f'(a)(x-a)$
This gives the equation of tangent line

$$f'(x) = \frac{d}{dx} e^{(-x^2-2x-1)}$$

$$= e^{(-x^2-2x-1)} \cdot (-2x-2)$$

Chain Rule Details
$\text{inner}(x) = -x^2-2x-1$
$\text{inner}'(x) = -2x-2$
$\text{outer}(\ ) = e^{(\ )}$
$\text{outer}'(\ ) = e^{(\ )}$

$$m = f'(0) = e^{(-(0)^2-2(0)-1)} \cdot (-2(0)-2)) =$$

$$= e^{-1} \cdot (-2) = \frac{1}{e^1} \cdot (-2) = \frac{-2}{e} \quad \left(\text{roughly } -\frac{2}{3} \text{ or } -1\right)$$

Common mistake

$$f'(x) = \cdots = \underbrace{e^{-x^2-2x-1}\cdot(-2x-2)}_{f'(x)} \underset{\text{not true!}}{=} \underbrace{e^{(-0^2-2(0)-1)}\cdot(-2(0)-2)}_{f'(0)}$$

Don't string together things that are not actually equal!

(b) Find the x coordinates of all points on graph of $f(x)$ that have horizontal tangent lines.

$m=0$

Solution Set $f'(x)=0$ and solve for x

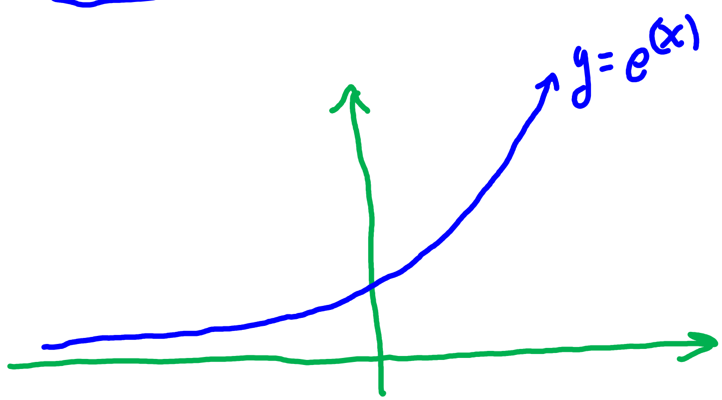
$$0 = m = f'(x) = \underbrace{e^{(-x^2-2x-1)}}_{\text{either this}=0} \cdot \underbrace{(-2x-2)}_{\text{or this}=0}$$

Remember

$\frac{a}{b}=0$ only when $a=0$ and $b\neq 0$

Zero Product Property

$a \cdot b=0$ only when $a=0$ or $b=0$ (or both)



$e^{(\text{anything})} > 0$

so the thing on the left cannot be 0.

So the thing on the right must be 0.

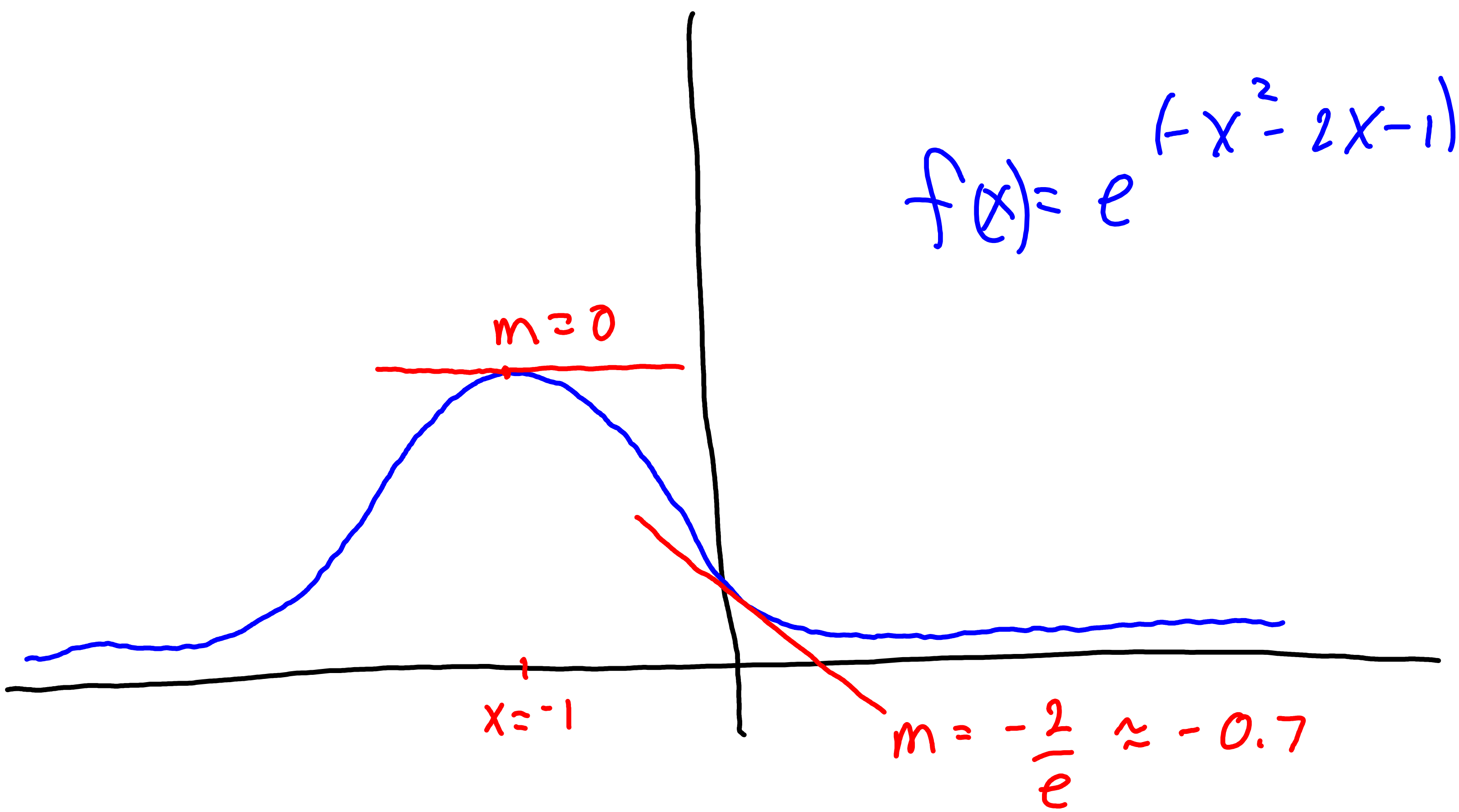
$-2x-2=0$ so $x=-1$

(c) Illustrate with a graph

Famous graph

$$y = e^{(2^{nd}\text{ degree polynomial with } \underline{\text{negative}} \text{ leading coefficient})}$$

have graphs that are Bell-Shaped Curves



Derivatives of Logarithmic Functions

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$ restricted to the domain $x > 0$

$$\frac{d}{dx}\ln(|x|) = \frac{1}{x}$$ true for all $x \neq 0$

$$\frac{d}{dx}\log_b(x) = \frac{1}{x \cdot \ln(b)}$$

[Example 3] $f(x) = \ln(x^2 - 3x + 17)$ find $f'(x)$

Solution

$$f'(x) = \frac{d}{dx} \ln(x^2 - 3x + 17)$$

Chain Rule

$inner(x) = x^2 - 3x + 17$

$inner'(x) = 2x - 3$

$outer(\ ) = \ln(\ )$

$outer'(\ ) = \frac{1}{(\ )}$

$$= \frac{1}{(x^2 - 3x + 17)} \cdot (2x - 3)$$

$$= \frac{2x - 3}{x^2 - 3x + 17}$$

[Example 4] $f(x) = \ln\left(\dfrac{\sqrt[3]{x^2-3x+17}}{(x^2+5)^{3/5}}\right)$ find $f'(x)$

Solution:

Since $f(x)$ has nested functions $\sqrt[3]{x^2-3x+17}$ and $(x^2+5)^{3/5}$ inside a quotient, inside the $\ln(\ )$ function, you might think that one would need to use the chain rule three times and the quotient rule. That would work, but would be really hard. Much easier to first rewrite $f(x)$.

$$f(x) = \ln\left(\sqrt[3]{x^2-3x+17}\right) - \ln\left((x^2+5)^{3/5}\right)$$

$$= \frac{1}{3}\ln(x^2-3x+17) - \frac{3}{5}\ln(x^2+5)$$

$\ln\frac{a}{b} = \ln(a) - \ln(b)$

$\ln(a^b) = b\ln(a)$

$$\text{So } f'(x) = \frac{1}{3}\underbrace{\frac{d}{dx}\ln(x^2-3x+17)}_{\text{use result of [Example 3]}} - \frac{3}{5}\underbrace{\frac{d}{dx}\ln(x^2+5)}$$

$$= \frac{1}{3}\cdot\overbrace{\frac{2x-3}{x^2-3x+17}} - \frac{3}{5}\cdot\overbrace{\frac{1}{(x^2+5)}\cdot 2x}$$

Chain Rule Details

$\text{inner}(x) = x^2+5$

$\text{inner}'(x) = 2x$

$\text{outer}(\ ) = \ln(\ )$

$\text{outer}'(\ ) = \dfrac{1}{(\ )}$

So $f'(x) = \dfrac{2x-3}{3(x^2-3x+17)} - \dfrac{6x}{5(x^2+5)}$

End of [Example 4]

$\boxed{\text{End of Lecture}}$