MATH 2301 (Barsamian) Lecture #21, Mon Oct 23, 2023

- Sign In
- Recitation Questions for tomorrow (Tue Oct 24) have been posted.
- Quiz Q6 This Fri Oct 27
- Quiz Q7 <u>Next</u> Fri Nov 3

Today, Tomorrow + Wednesday

Section 4.1 Maximum and Minimum Values

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Definition of Absolute Max (and Min) Value of a function on a domain D.

Words The Absolute Max (Min) Value of function f on domain D.

Meaning a y value $y = f(c)$ such that

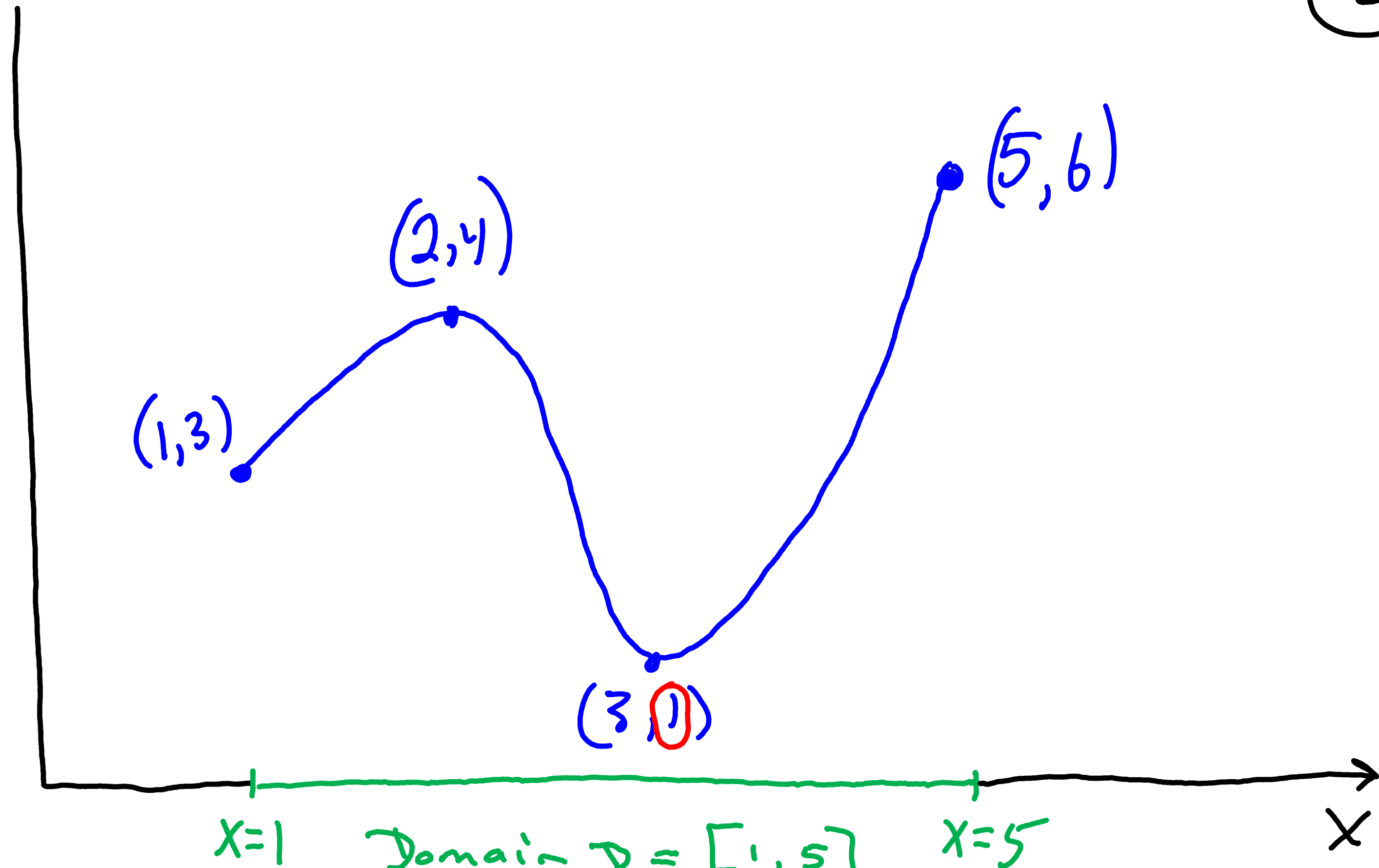
- $c \in D$

  is an element of

- $f(c) \geq f(x)$ for all $x \in D$

  (for Absolute Min, $f(c) \leq f(x)$)

[Example 1]



Absolute Max value is $y=6$ (it occurs at $x=5$)

Or could write Abs Max Value is $y=6=f(5)$

indicates where the max value occurs.

Absolute Min Value is $y=1=f(3)$

Observe Abs Max or Min may occur at endpoints of the domain D

Definition of Local Max (and Local Min)

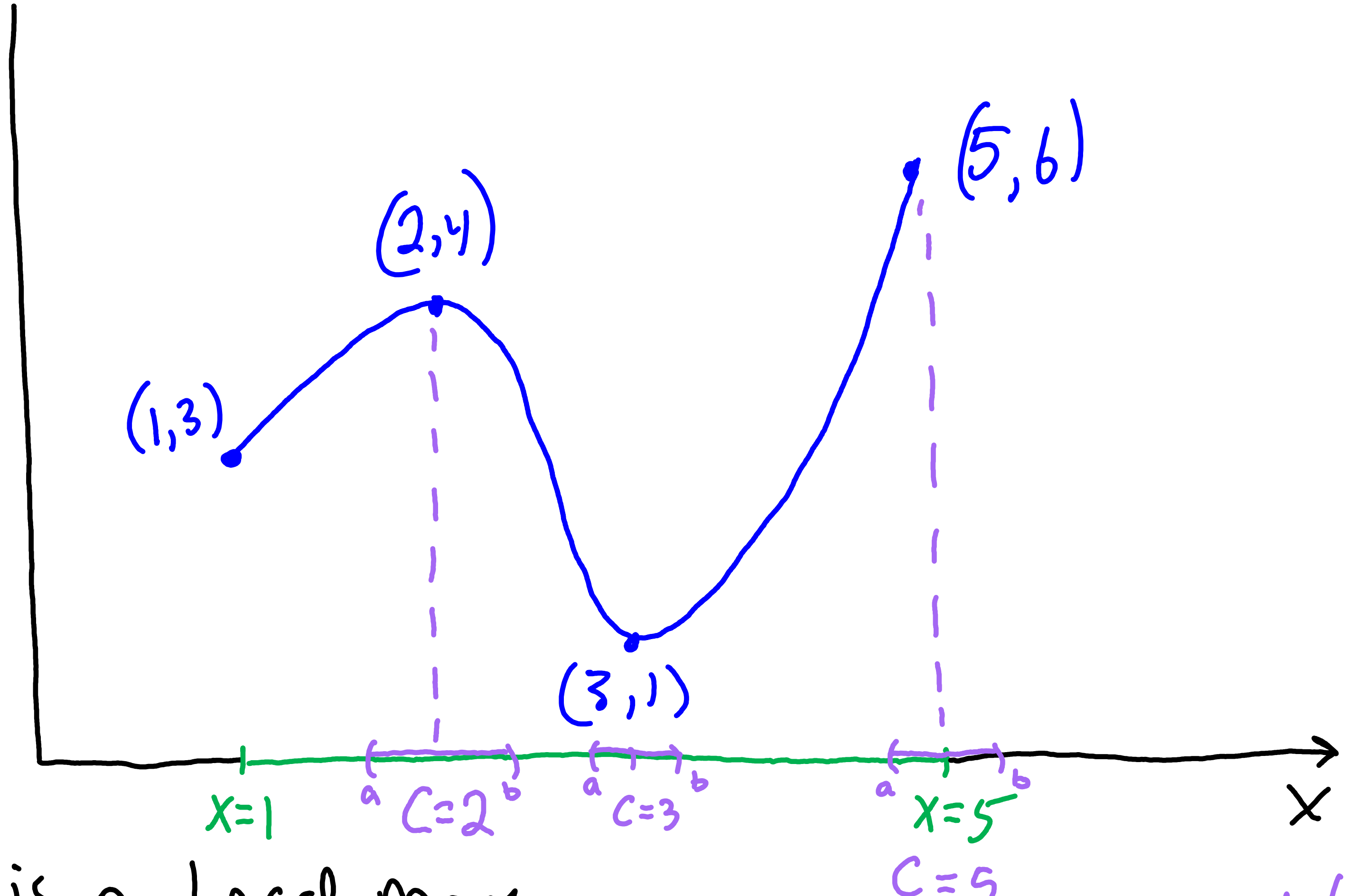
Words: The local maximum (local minimum) value of f on Domain D

meaning: A y value $y = f(c)$ such that

- $c \in D$ is an element of
- There is an open interval $(a,b) \subset D$ is a subset of such that $c \in (a,b)$. That is $a < c < b$ and such that $f(c) \geq f(x)$ for all x in (a,b)

($f(c) \leq f(x)$ for all x in (a,b) for the local min value)

[Example 2]



$y = 4 = f(2)$ is a Local max

$y = 6 = f(5)$ is NOT a Local max ⟶ { any open interval (a, b) containing $c = 5$ will not be contained in D }

$y = 1 = f(3)$ is a local min

$y = 3 = f(1)$ is NOT a Local min

Local max & min cannot occur at endpoints!

If $f(x)$ is described by a graph, the Abs max & min values and Local max & min values are easy to spot.

But what about functions described by a formula, not by a graph?

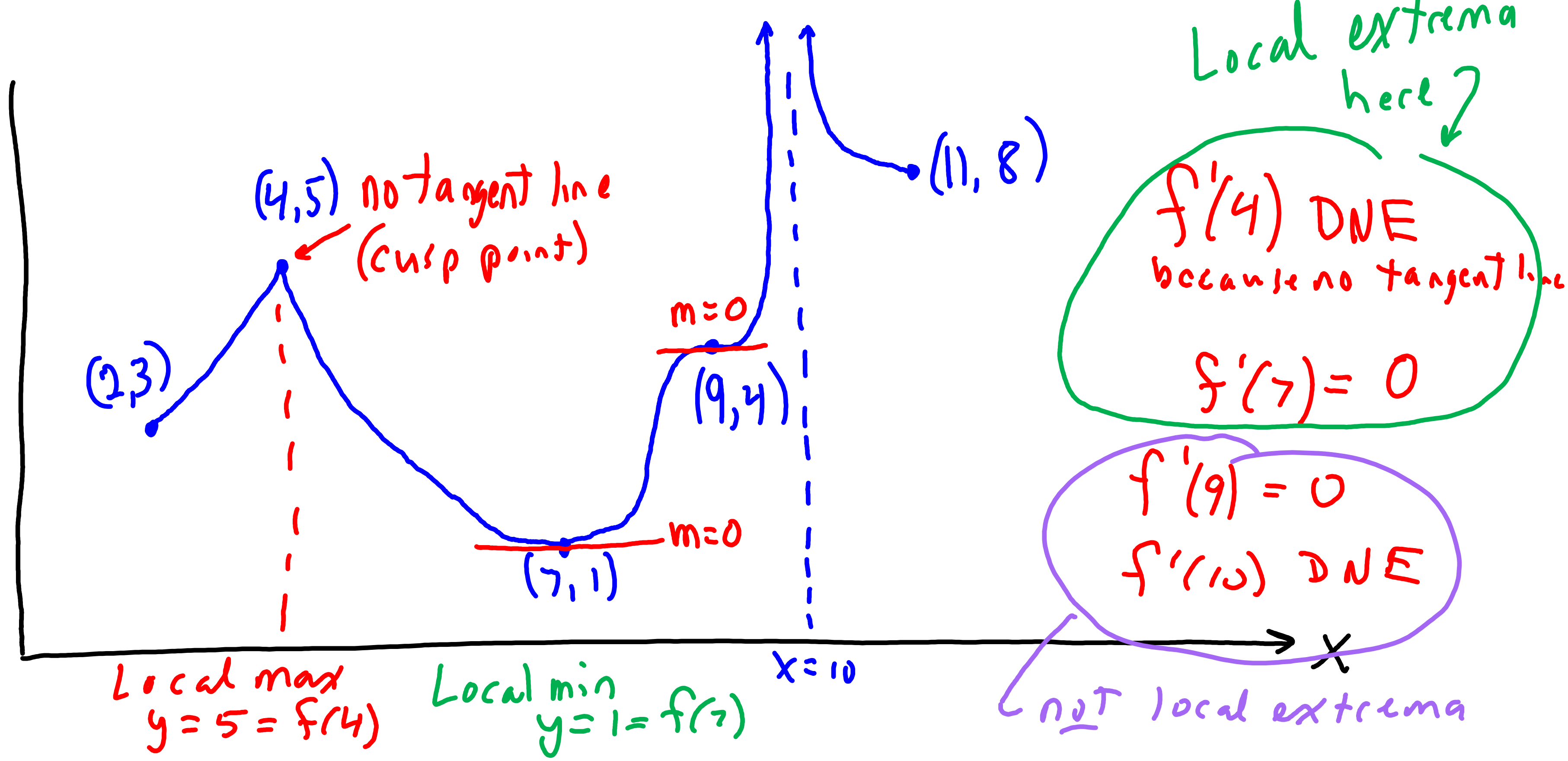
We seek: Methods for analyzing the formula for a function f to determine max and min values.

Start With Local Max & Min

Fermat's Theorem

If $f(c)$ is a local max or min value, then $f'(c) = 0$ or $f'(c)$ does not exist

Picture to illustrate that this is believable



Definition of Critical Number

Words: Critical Number for a function f

Meaning: an x value $x=c$ such that

- $f'(c)=0$ or $f'(c)$ Does not exist
- $x=c$ is in the domain of f.
  (That is $f(c)$ exists)

Observe local extrema can only occur at x coordinates that are critical numbers (and are not endpoints)

[Example 3] Let $f(x) = x^4 e^{(-5x)}$

Find all critical numbers of $f(x)$

Solution

Note: the Domain of $f(x)$ is all real numbers

We have to investigate $f'(x)$

$$f'(x) = \frac{d}{dx}\Big(\underbrace{x^4 e^{(-5x)}}_{\text{product}}\Big)$$

$$= \left(\frac{d}{dx}x^4\right)e^{(-5x)} + x^4 \frac{d}{dx}\underbrace{e^{(-5x)}}_{\text{nested}}$$

chain rule

$$= 4x^3 e^{-5x} + x^4 \cdot \left((-5)\, e^{(-5x)}\right)$$

$$= \underbrace{e^{(-5x)}}_{\text{this term is never } 0} \cdot \underbrace{x^3}_{\text{this term is 0 when } x=0} \cdot \underbrace{(4 - 5x)}_{\text{this term is 0 when } x = 4/5}$$ factored version

Observe: Domain of $f'(x)$ is all real numbers.

There are no x values that cause $f'(x)$ to not exist.

$f'(x) = 0$ when $x = 0$ or $x = \frac{4}{5}$

Since Domain of $f(x)$ is all real numbers we know $f(x)$ exists for these x values.

So the critical numbers are

$x = 0$ and $x = 4/5$

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end of lecture