MATH 2301 (Barsamian) Lecture #22 (Wed Oct 25, 2023) ①

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Upcoming Quiz Q6 this Fri Oct 27
Quiz Q7 next Fri Nov 3

Today: More discussion of Section 4.1 Absolute Max + Min Values

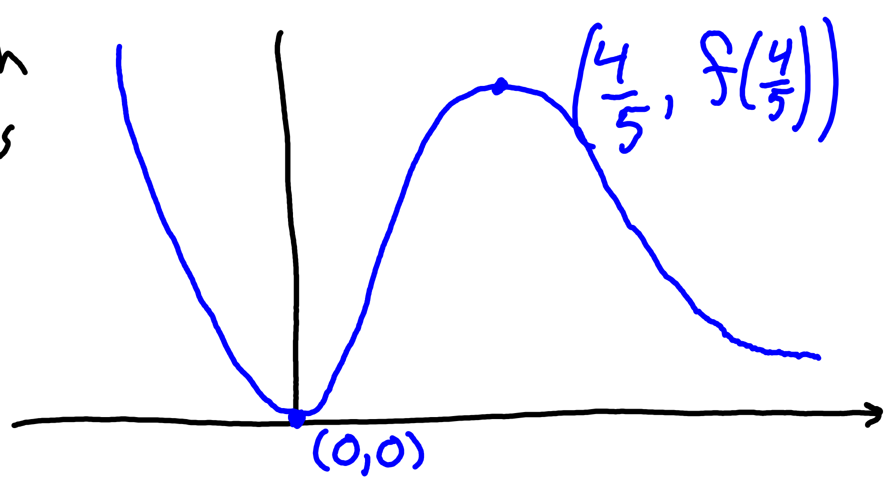
Remember: We're on a quest: formulate analytical methods for determining extreme values (max or min) for a function $f(x)$ that is given by a formula

Important Concept: Critical Numbers for a function $f(x)$

We did [example] find critical numbers of $f(x) = x^4 e^{(-5x)}$

we found critical numbers $x=0$ and $x = 4/5$

Confirm with graph from Desmos



Local Max $y = f\left(\frac{4}{5}\right)$

Local Min $y = 0 = f(0)$

Meeting Part 1 More about Critical Numbers

Find critical numbers of $f(x) = x + \frac{9}{x}$

Solution

Start by finding $f'(x)$

First rewrite $f(x) = x + \frac{9}{x} = x + 9\left(\frac{1}{x}\right) = \underline{x + 9x^{-1}}$

power function form

So $f'(x) = \frac{d}{dx}\left(x + 9x^{-1}\right) = 1 + 9(-1)x^{-1-1} = 1 - 9x^{-2}$

$= 1 - 9\left(\frac{1}{x^2}\right) = 1 - \frac{9}{x^2} = f'(x)$

X values that cause $f'(x)$ to not exist:

$x = 0$, because $f'(0) = 1 - \frac{9}{(0)^2} = 1 - \frac{9}{0}$ DNE

X values that cause $f'(x) = 0$:

$0 = f'(x) = 1 - \frac{9}{x^2}$

$\frac{9}{x^2} = 1$

$9 = x^2$ Solutions: $x = 3$ and $x = -3$

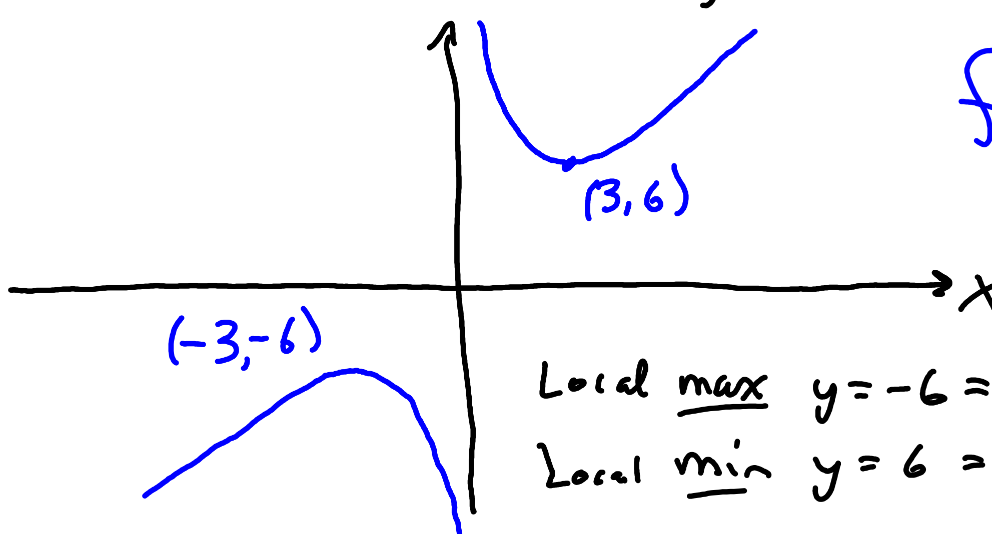
So our candidate x values are $x=0,\ x=-3,\ x=3$

Must check the other condition. Does $f(x)$ exist at these x values?

x	$f(x) = x + \frac{9}{x}$
0	$f(0) = 0 + \frac{9}{0}$ DNE
-3	$f(-3) = -3 + \frac{9}{-3} = -3 + (-3) = -6$ exists ✓
3	$f(3) = 3 + \frac{9}{3} = 3 + 3 = 6$ exists ✓

So the critical numbers are $x=-3,\ x=3$

Confirm with graph from Desmos



$f(x) = x + \frac{9}{x}$

Local max $y = -6 = f(-3)$

Local min $y = 6 = f(3)$

end of example

Meeting Part 2 Finding Absolute Max & Min Values with the Closed Interval Method

Fact. Absolute Max & Min Values can only occur at x values that are

- endpoints of the domain

or

- critical numbers in the domain

Extreme Value Theorem

If $f(x)$ is continuous on a closed Interval $[a,b]$, then $f(x)$ attains both an abs max value y_{max} and an absolute min value y_{min} on that interval

These two facts enable us to formulate a method for finding abs max & min values in certain situations.

The Closed Interval Method

for finding Abs max value and Abs min value for a continuous function on a closed interval.

Step 1: Confirm that domain is a closed interval and that $f(x)$ is continuous there.

Step 2: Find critical numbers of $f(x)$

Step 3: Make a 2-column table.

List Here. x values in increasing order

important x values	corresponding y values
$x = a$ endpoint	$y = f(a)$
$x = c_1$ critical	$y = f(c_1)$
⋮ Critical numbers in the interval	⋮
$x = c_k$ critical	$y = f(c_k)$
$x = b$ endpoint	$y = f(b)$

Step 4: Identify the greatest y value, y_{max}, and the least y value, y_{min}, and write a clear conclusion

[Example] Find Abs Max + Min Values for the function

$$f(x) = -x^4 + 6x^2 + 27 \text{ on the interval } [-2, 3]$$

Solution

Step 1 $[-2, 3]$ is a closed interval ✓

f is continuous on the interval because f is polynomial ✓

Step 2 Find critical numbers of $f(x)$

Start with $f'(x) = -4x^3 + 12x$

Observe f' is polynomial, so $f'(x)$ always exists.

Set $f'(x) = 0$ and solve for x

$$0 = f'(x) = -4x^3 + 12x = -4x(x^2 - 3)$$

$$p^2 - q^2 = (p+q)(p-q)$$

$$= -4x(x + \sqrt{3})(x - \sqrt{3})$$

Solutions are $x = 0$, $x = -\sqrt{3}$, $x = \sqrt{3}$

These are the only candidates to be critical numbers

We have to check the other criterion:

does $f(c)$ exist?

We already know $f(x)$ always exists, because f polynomial.

So $x=-\sqrt{3}$, $x=0$, $x=\sqrt{3}$ are the critical numbers

Step 3 Table

important x values	important y values $y=f(x)=-x^4+6x^2+27$
$x=-2$ (endpoint)	$y=f(-2)=35$
$x=-\sqrt{3}$ (crit)	$y=f(-\sqrt{3})=\cdots=36$ y_{max}
$x=0$ (crit)	$y=f(0)=27$
$x=\sqrt{3}$ (crit)	$y=f(\sqrt{3})=\cdots=36$ y_{max}
$x=3$ (endpoint)	$y=f(3)=\cdots=0$ y_{min}

Step 4 The absolute max value is $y_{max}=36=f(-\sqrt{3})=f(\sqrt{3})$

The absolute min value is $y_{min}=0=f(3)$

End of Example

End of Lecture