Sign In

Quiz Q6 Today

Graded Exams + Quizzes will be returned on Monday

Quiz Q7 Next Friday, Nov 3

Today: Section 4.2 The Mean Value Theorem

Section 4.2 Meeting Part 1

Rolle's Theorem

If function f satisfies these criteria

f is continuous on the closed interval $[a,b]$

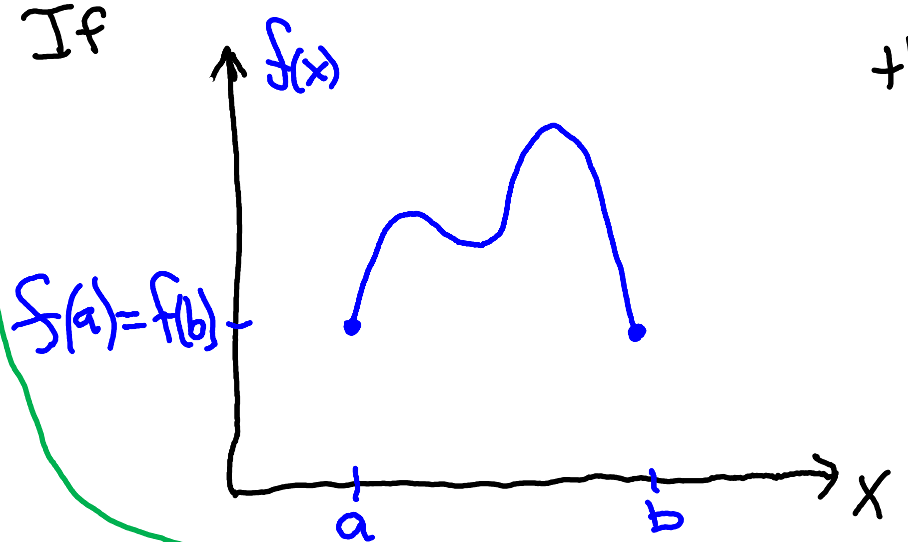
f is differentiable on the open interval (a,b)

$f'(x)$ exists

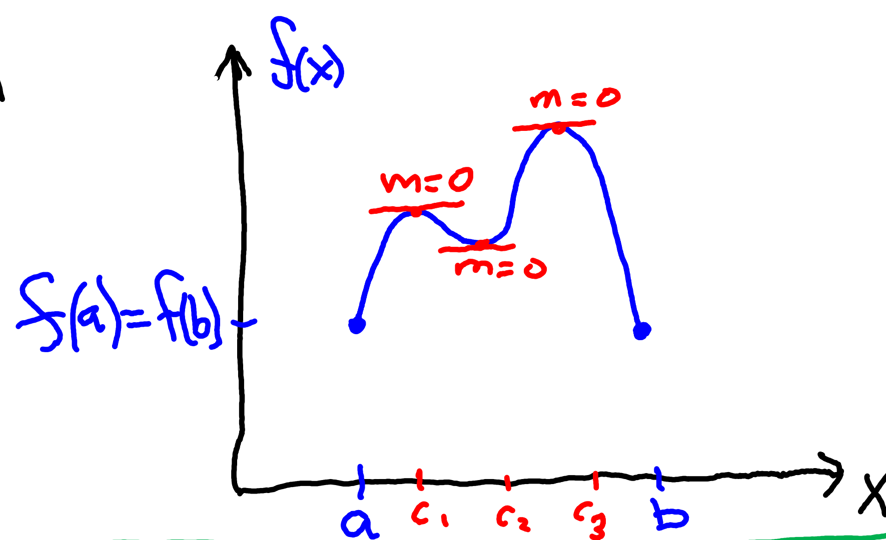
$f(a) = f(b)$

Then there exists an $x = c$ (at least one) in the open interval (a,b) such that $f'(c) = 0$

If



then



[Example 1] (a) Show that for the function $f(x)=\sqrt{x}-\frac{1}{3}x$, there is at least one x value on the interval $(0,9)$ where $f'(x)=0$.

Solution: Use Rolle's Theorem

Start by showing that all the hypotheses (the criteria) are satisfied

Continuity

The function $y=\frac{1}{3}x$ is polynomial, so always continuous

the function $y=\sqrt{x}$ is continuous on whole interval $[0,\infty)$

So f is certainly continuous on closed interval $[0,9]$

Differentiability

$$f'(x)=\frac{d}{dx}\left(\sqrt{x}-\frac{1}{3}x\right)=\frac{d}{dx}x^{1/2}+\frac{1}{3}\frac{d}{dx}x=\cdots=\frac{1}{2\sqrt{x}}-\frac{1}{3}$$

Domain of f' is the open interval $(0,\infty)$

So certainly f is differentiable on the open interval $(0,9)$

Endpoints

$$f(0)=\sqrt{0}-\frac{1}{3}(0)=0$$

$$f(9)=\sqrt{9}-\frac{1}{3}(9)=3-3=0$$

these match ✓

Conclusion There exists at least one $x=c$ in the open interval $(0,9)$ such that $f'(c)=0$

(b) Find all the $x=c$ that satisfy the conclusion of Rolle's Theorem (4)

Solution

$$f'(c)=0$$

$$\frac{1}{2\sqrt{c}}-\frac{1}{3}=0$$

$$\frac{1}{2\sqrt{c}}=\frac{1}{3}$$

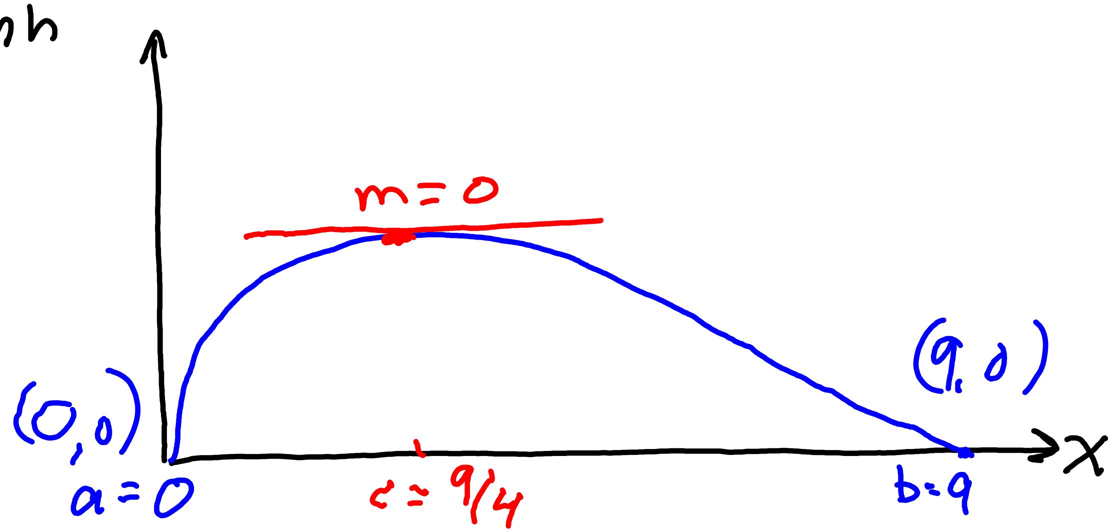
$$2\sqrt{c}=3$$

$$\sqrt{c}=\frac{3}{2}$$

$$c=\left(\frac{3}{2}\right)^2=\frac{9}{4}$$

(c) Illustrate with a graph

from Desmos



end of example

The Mean Value Theorem (MVT)

If $f(x)$ satisfies these criteria (the hypotheses):

f is continuous on closed interval $[a, b]$

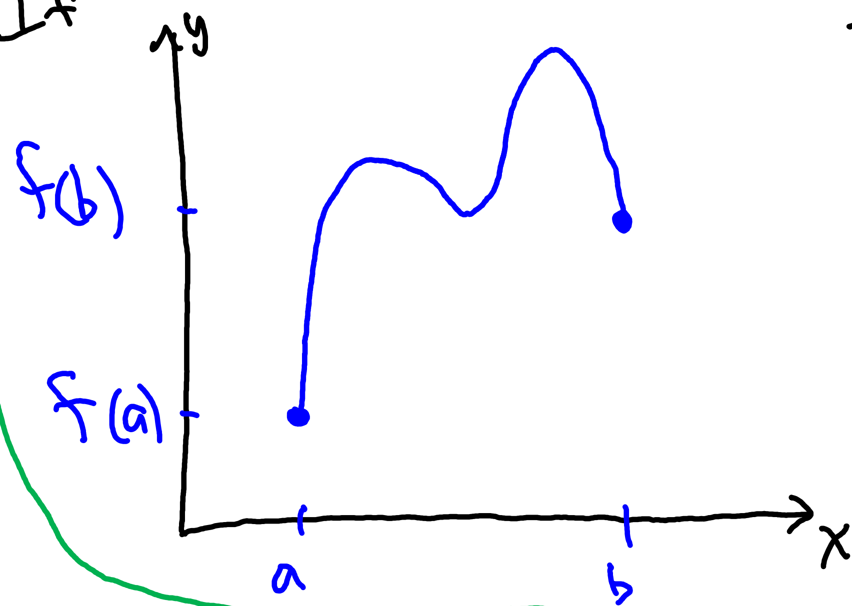
f is differentiable on open interval (a, b)

Then this statement is true (the conclusion):

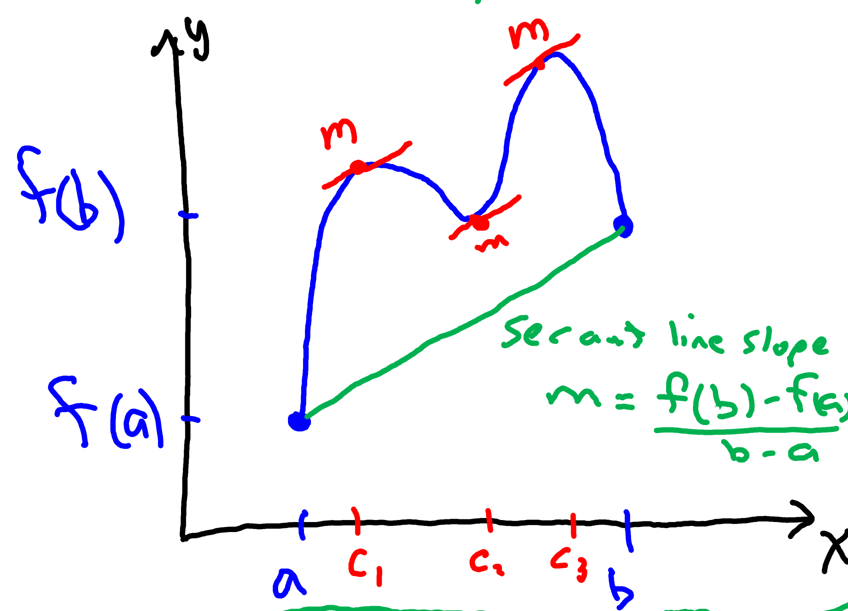
There is a number $x = c$ (at least one) in the open interval (a, b)

Such that $$\underbrace{f'(c)}_{\text{tangent line slope}} = \underbrace{\frac{f(b) - f(a)}{b - a}}_{\text{secant line slope}}$$

If



then



[Example] Let $f(x) = \sqrt{x}$

(a) Show that $f(x)$ satisfies hypotheses of Mean Value Theorem on the closed interval $[0,4]$

<u>Solution</u>

f is <u>continuous</u> on the closed interval $[0,4]$

$f'(x) = \frac{1}{2\sqrt{x}}$ exists on open interval $(0,\infty)$

So f <u>is</u> differentiable on the open interval $(0,4)$

(b) Write the <u>conclusion</u> of the Mean Value Theorem

<u>Solution</u> There exists at least one $x = c$ in the open interval $(0,4)$

Such that $f'(c) = \frac{f(4) - f(0)}{4 - 0}$

(c) Find all the c that satisfy this conclusion

<u>Solution</u>

$$f'(c) = \frac{\sqrt{4} - \sqrt{0}}{4 - 0} = \frac{2}{4} = \frac{1}{2}$$

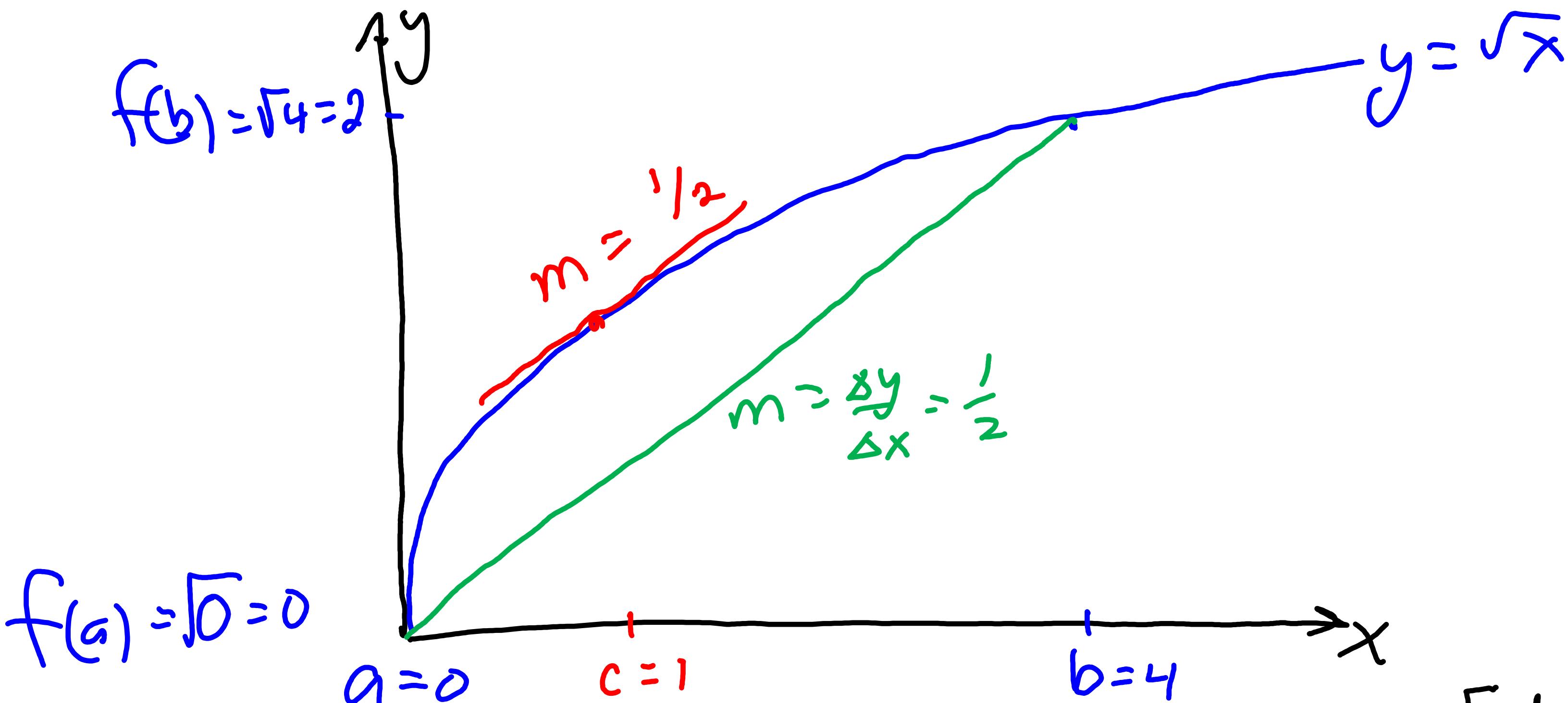
$$\frac{1}{2\sqrt{c}} = \frac{1}{2}$$

$$2\sqrt{c} = 2$$

$$\sqrt{c} = 1$$

$$c = 1$$

(d) Illustrate with a graph



End of Example

End of Lecture