MATH 2301 (Barsamian) Lecture #29 (Wed Nov 15)

Today: 5.1

Friday: 5.2

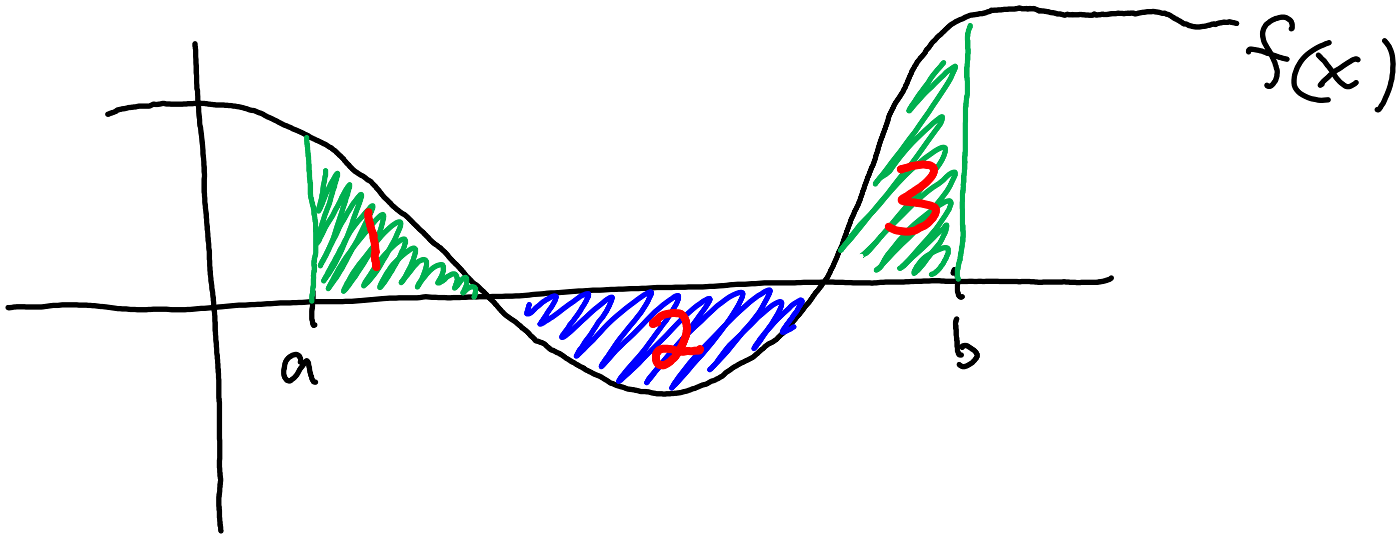
Mon: 5.3 Quiz Q8

Tues: Recitation

Section 5.1 Areas + Distances

Meeting Part 1: Areas

Two kinds of area between function $f(x)$ and x axis from $x=a$ to $x=b$.



Unsigned Area (USA): All regions have positive area. USA = ① + ② + ③

Signed Area (SA): regions below x axis get a minus sign

$$SA = ① - ② + ③$$

The Area Problem

For a general curvy function $f(x)$ whose graph is not made up of basic geometric shapes,

(1) What do we even mean by "Area"?

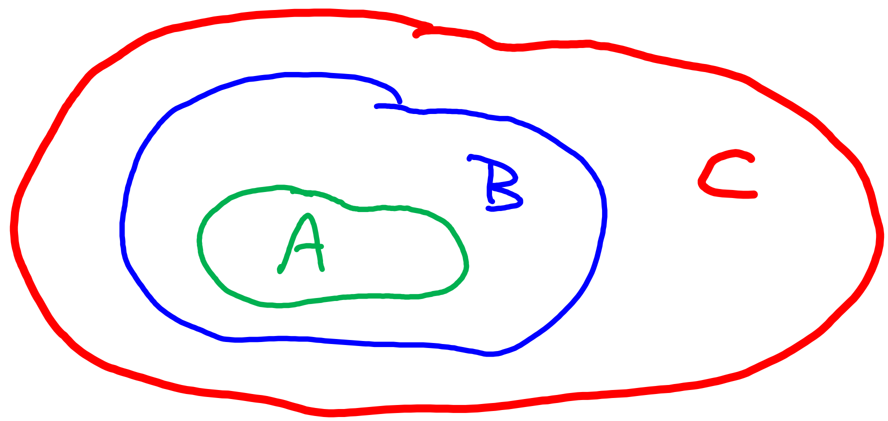
(2) How do we compute a value for the Area?

Try to answer question (2) first (oddly)

We expect Area (whatever it means) to behave this way

If

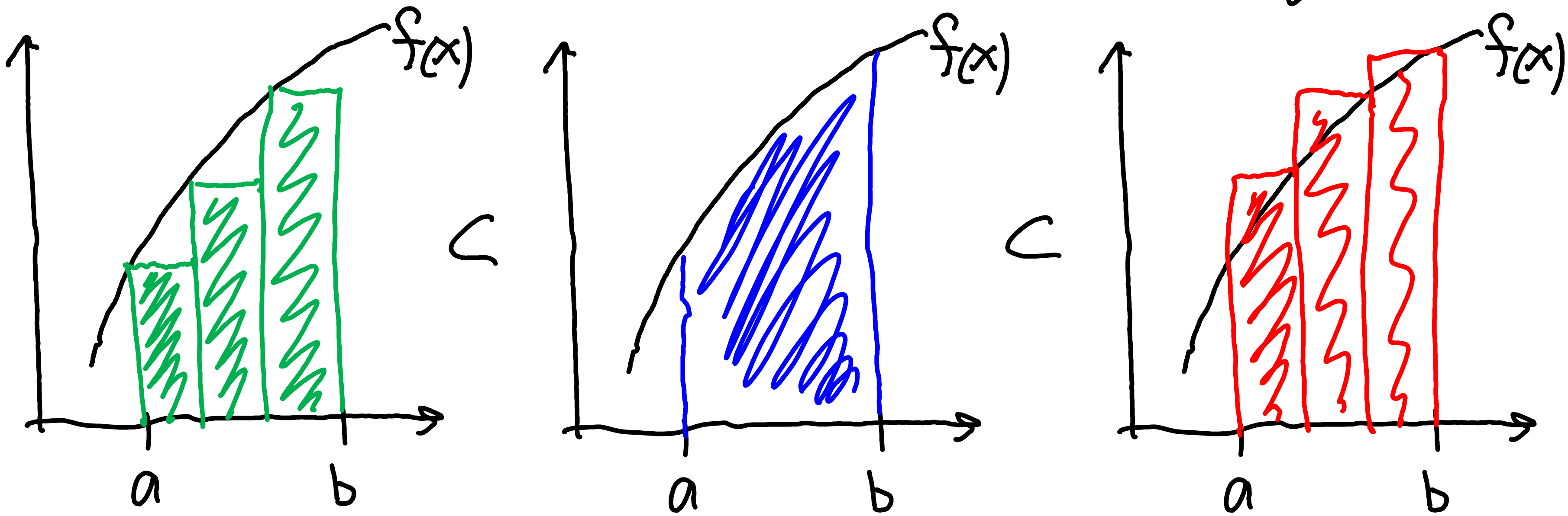
Region A $\subset$ Region B $\subset$ Region C



then we expect that

Area A $<$ Area B $<$ Area C

Sometimes we can create a "Sandwich" of regions



Region A — unknown area Region B — Region C

We expect

Area A < Unknown Area B < Area C

Riemann Sums

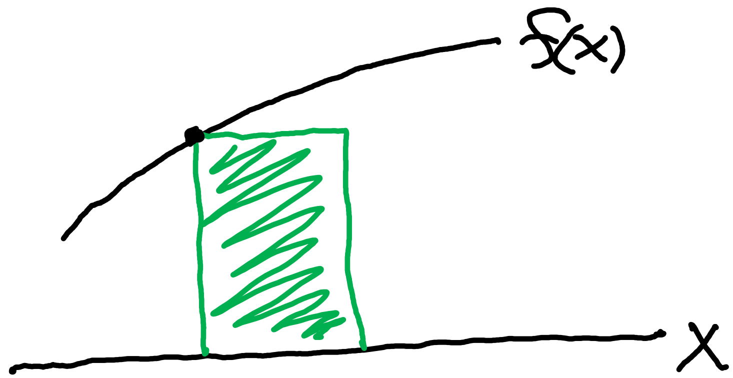
Definition of Left Sum

Symbol: L_n

Spoken: Left Sum with n rectangles

Usage: There is some continuous function f(x) in the discussion and an interval [a, b]

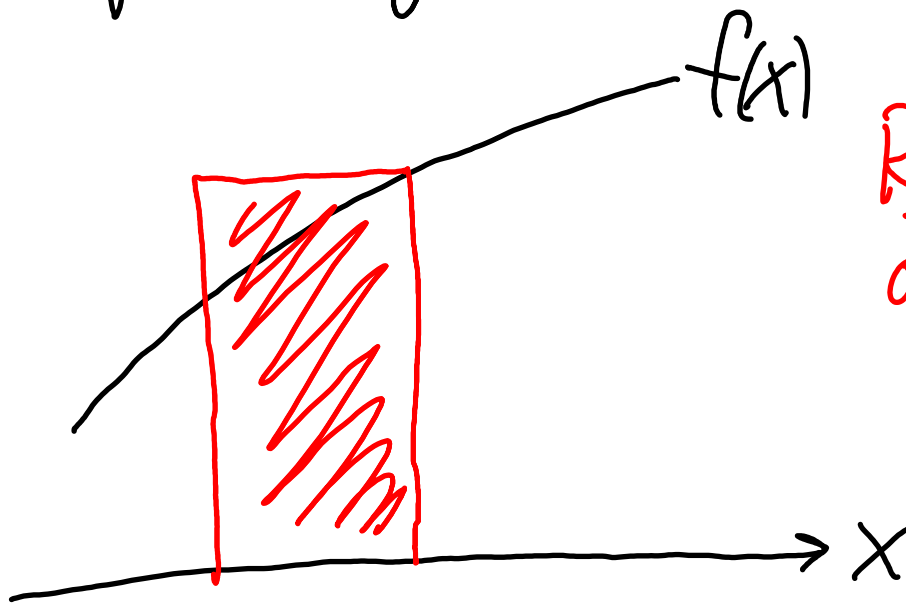
Meaning: the sum of the areas of n equal width "Left Rectangles" that are parked on interval [a, b]



Rectangle's left corner is on graph of f(x)

Similarly for Right Sum, R_n

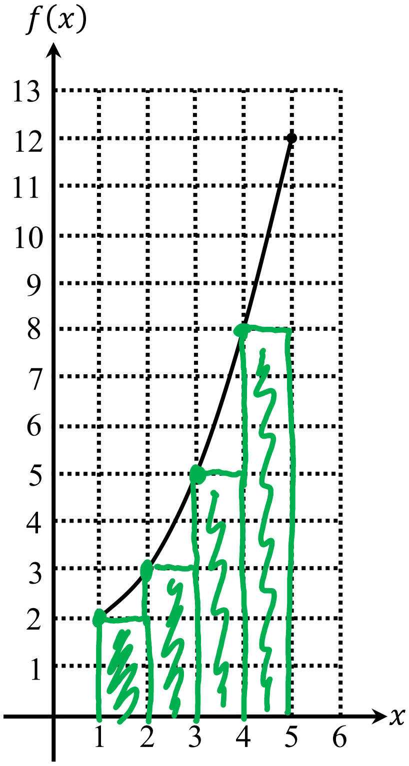
Uses n "Right Rectangles"



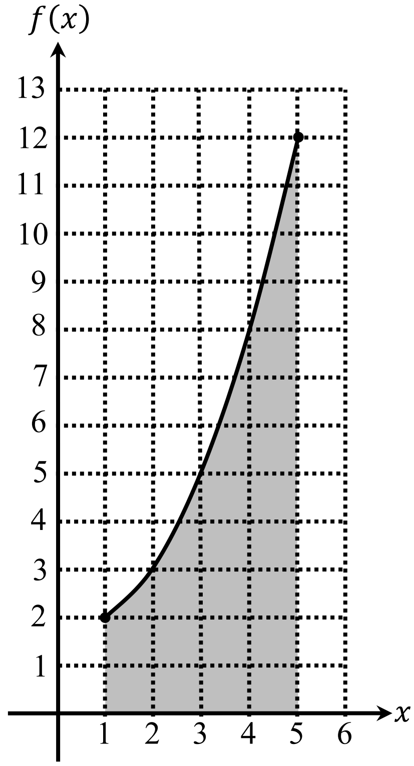
Right corner is on the graph of f(x)

Class Drill: Estimating the Area Under a Graph by Using Riemann Sums

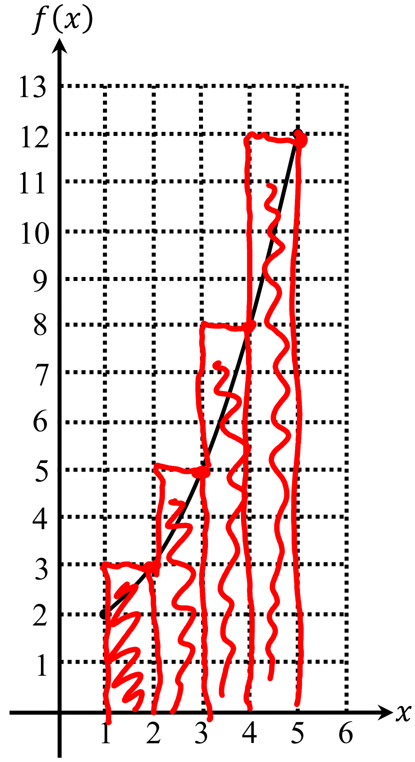
The goal is to estimate the shaded area in the middle figure. You will do this by finding the values of the Riemann sums L_4 and R_4. This will give you lower and upper bounds for the shaded area.



L_4



unknown area



R_4

(A) Draw in the rectangles for the left sum L_4.

(B) Find the value of L_4. $L_4 = 2 \cdot 1 + 3 \cdot 1 + 5 \cdot 1 + 8 \cdot 1 = 18$

(C) Draw in the rectangles for the right sum R_4.

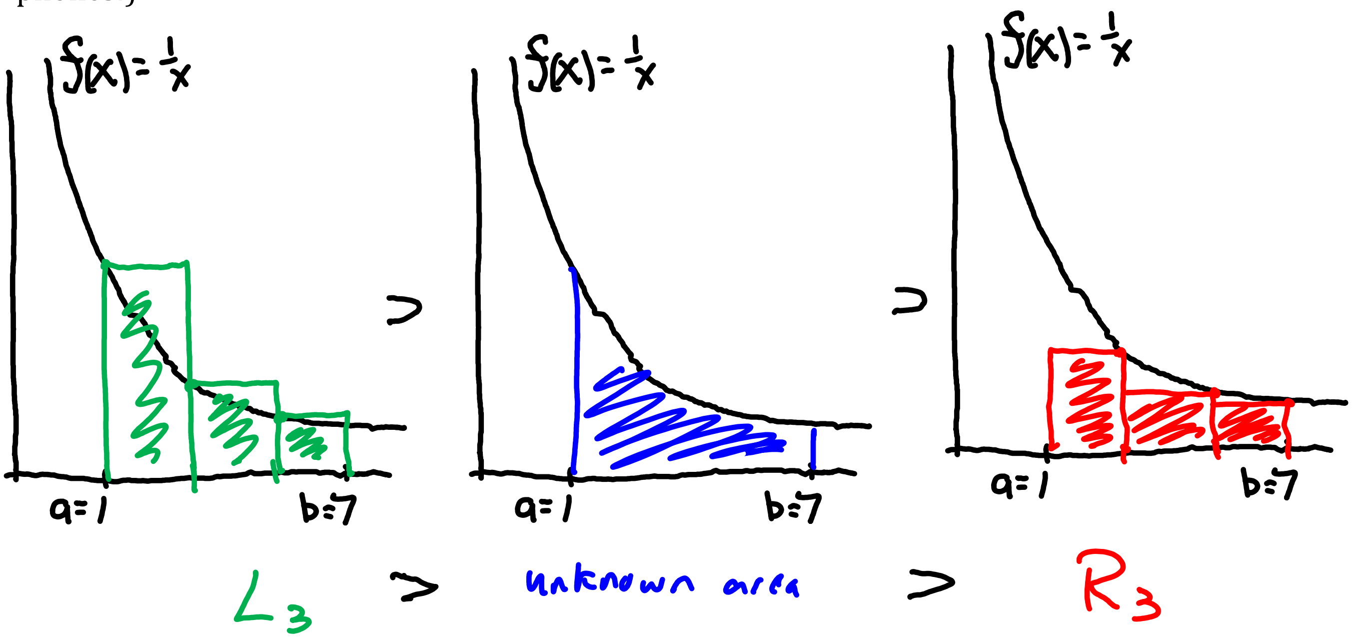
(D) Find the value of R_4. $R_4 = 3 \cdot 1 + 5 \cdot 1 + 8 \cdot 1 + 12 \cdot 1 = 28$

(E) Use the values from questions (B)and (D) to build a true inequality

18 < **unknown area** < 28

Class Drill: Computing Riemann Sums

The goal is to find approximations for the signed area between the graph of the function $f(x) = \frac{1}{x}$ and the x axis on the interval [1,7] by computing Left and Right Riemann Sums with 3 rectangles. That is, find values for L_3 and R_3. Show all details clearly. (Hand calculations! No calculators or cell phones!)



$$\text{Rectangle Width} = \Delta x = \frac{b-a}{3} = \frac{7-1}{3} = \frac{6}{3} = 2$$

Important X values

$X_0 = a = 1$ $\quad X_1 = X_0 + \Delta x = 1+2 = 3$ $\quad X_2 = X_1 + \Delta x = 3+2 = 5$ $\quad X_3 = b = 7$

$$L_3 = f(x_0)\cdot\Delta x + f(x_1)\Delta x + f(x_2)\Delta x$$

$$= \frac{1}{x_0}\cdot\Delta x + \frac{1}{x_1}\Delta x + \frac{1}{x_2}\Delta x$$

(10)

$$= \left(\frac{1}{x_0} + \frac{1}{x_1} + \frac{1}{x_2}\right)\Delta x$$

$$= \left(\frac{1}{1} + \frac{1}{3} + \frac{1}{5}\right)\cdot 2$$

$$= \left(\frac{15}{15} + \frac{5}{15} + \frac{3}{15}\right)\cdot 2$$

$$= \frac{23}{15}\cdot 2$$

$$= \frac{46}{15}$$

$$R_3 = \left(f(x_1) + f(x_2) + f(x_3)\right) \cdot \Delta x$$

$$= \left(\frac{1}{3} + \frac{1}{5} + \frac{1}{7}\right) \cdot 2$$

$$= \left(\frac{35}{105} + \frac{21}{105} + \frac{15}{105}\right) 2$$

$$= \frac{71}{105} \cdot 2$$

$$= \frac{142}{105}$$

$L_3 = \frac{46}{15}$ > unknown area > $\frac{142}{105}$

roughly 3 > unknown area > roughly $\frac{3}{2}$

end of Class Drill

End of Lecture