Sign In

Today Section 5.2

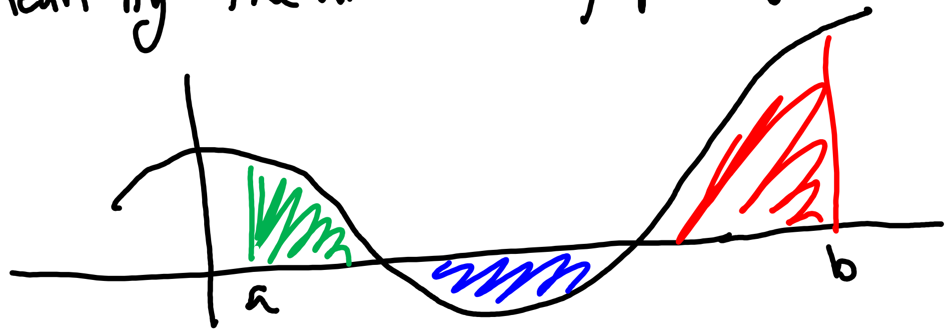
Monday Lecture Section 5.3, Quiz Q8

Tuesday Recitation Sections 5.1, 5.2, 5.3

Recall from Wednesday

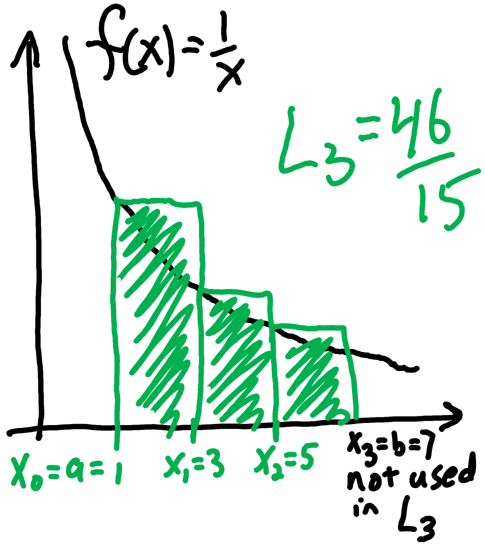
The Area Question for general (curvy) function

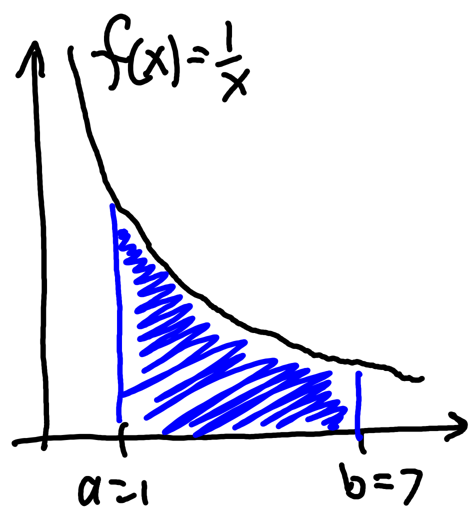
- What do we mean by "the area between graph of f(x) and x axis from x=a to x=b"?

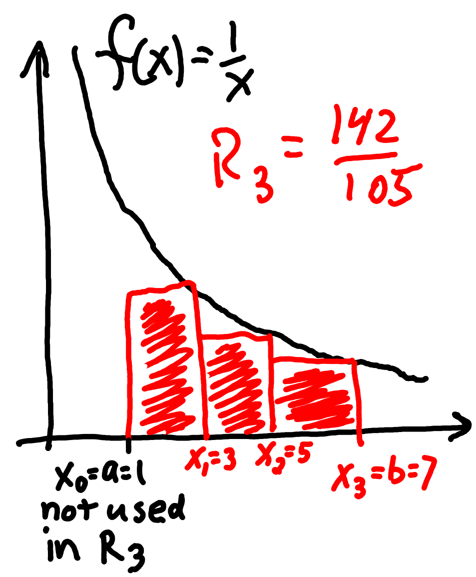


- How do we compute its value?

We worked on approximating the unknown area (To answer 2nd Question)



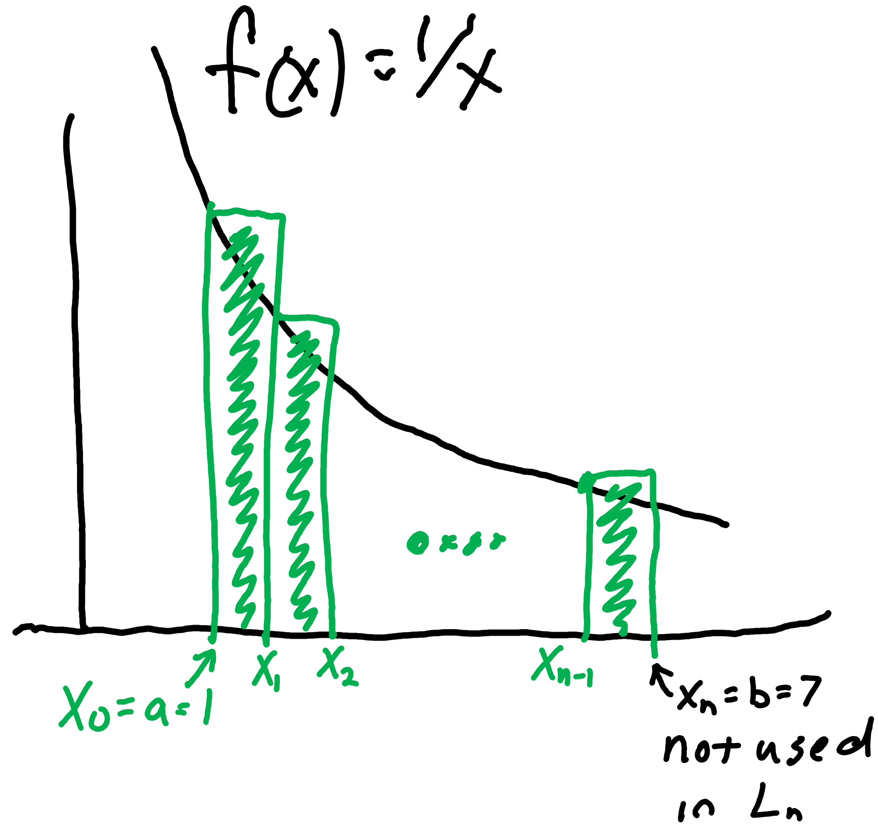




$$L_3 = f(x_0)\Delta x + f(x_1)\Delta x + f(x_2)\Delta x$$

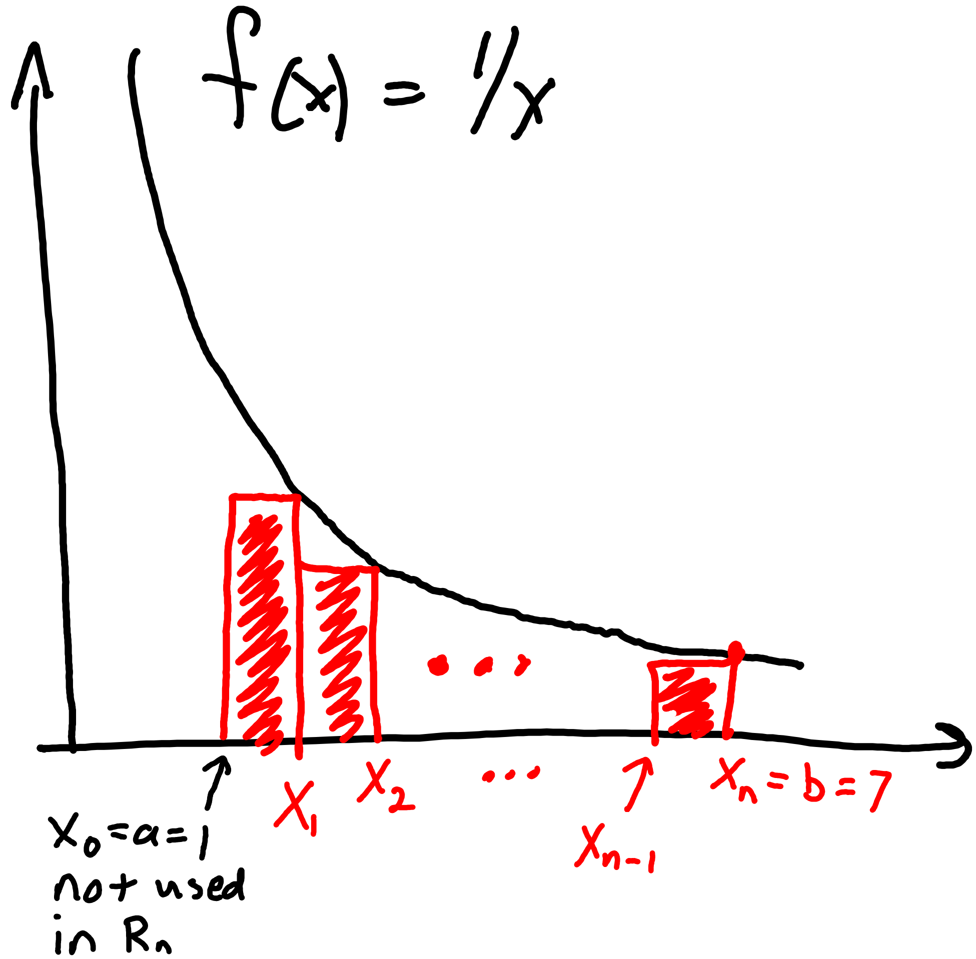
$$= \sum_{i=0}^{i=2} f(x_i)\Delta x \quad \text{Summation notation}$$

More generally



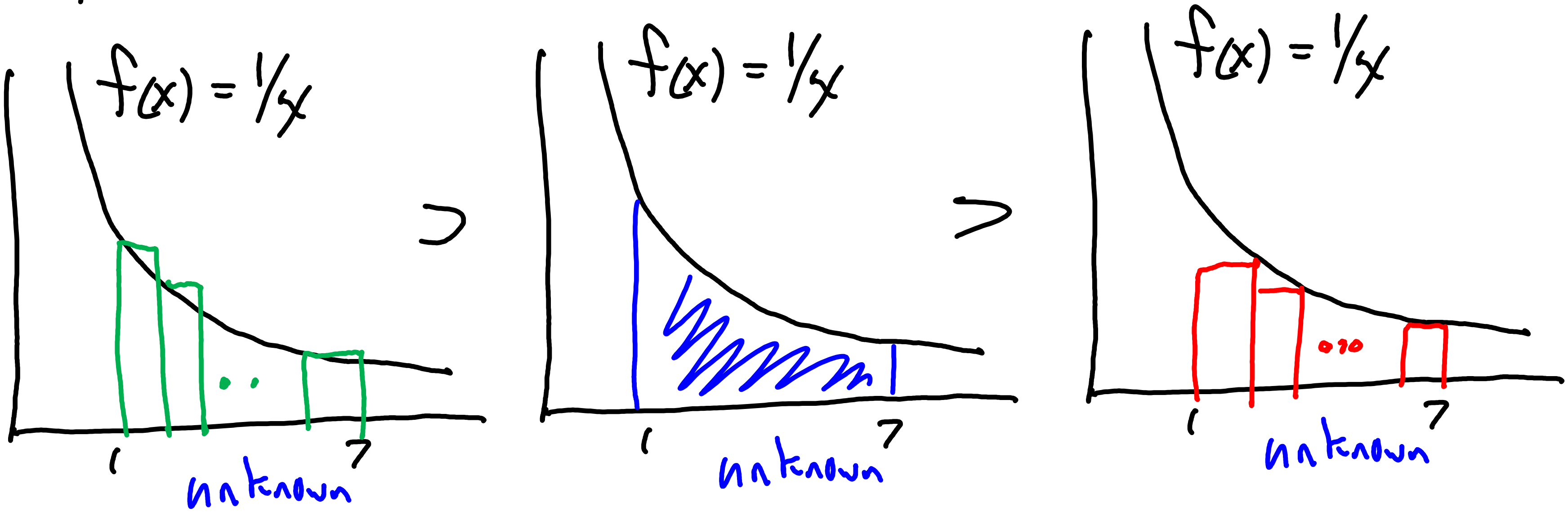
$$L_n = \sum_{i=0}^{i=n-1} f(x_i)\Delta x$$

Similarly for Right Sums



$$R_n = \sum_{i=1}^{i=n} f(x_i)\Delta x$$

Try to get better estimate of the unknown area by using more rectangles. 5



by using more & more rectangles

n	L_n	unknown area	R_n
10	$L_{10} \approx 2.2315$	?	$R_{10} \approx 1.7172$
100	$L_{100} \approx 1.9719$	?	$R_{100} \approx 1.9205$
1000	$L_{1000} \approx 1.9485$	?	$R_{1000} \approx 1.9433$

$n \to \infty$

estimate unknown area ≈ 1.946

Observe

It looks like $\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n \approx 1.946$

Huge Fact

If $f(x)$ is continuous on interval $[a, b]$ (except for possibly a finite number of discontinuities)

Then the $\lim_{n\to\infty} L_n$ and $\lim_{n\to\infty} R_n$ both exist and have the same value

$$\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$$

The Huge Fact enables us to make this <u>Definition</u>

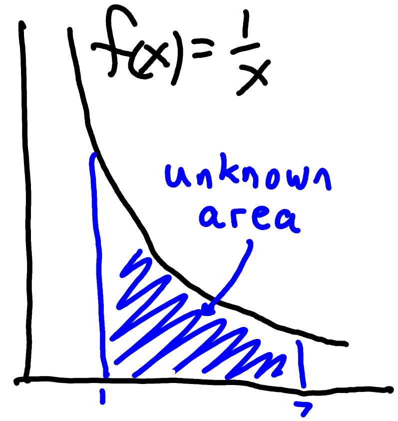
(which answers the <u>Area Question</u> posed earlier)

Definition of Definite Integral

- <u>Symbol</u>: $\displaystyle\int_{x=a}^{x=b} f(x)\,dx$
- <u>Spoken</u>: The definite integral of f from a to b.
- <u>Alternate Terminology</u>: The <u>signed area</u> between $f(x)$ and x axis from $x=a$ to $x=b$.
- <u>Usage</u> $f(x)$ is continuous on interval $[a,b]$ (except for possibly a finite number of discontinuities)
- <u>meaning</u> The number $\displaystyle\lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n$

Observe that this definition answers <u>both parts</u> of the Area Question

[Example] We found



≈ 1.9486

Write this result using Definite Integral notation.

Solution:

$$\underbrace{\int_{x=1}^{x=7} \frac{1}{x}\, dx}_{\text{exact symbol for the blue area}} \approx \underbrace{1.9486}_{\text{decimal approximation that we found}}$$

Why the funky notation?

$$\int_{x=a}^{x=b} f(x)\,dx = \lim_{n\to\infty} R_n$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x$$

dx is a remnant of the Δx symbol

$\int_a^b$ is a remnant of the $\sum_{i=1}^{i=n}$ symbol

Example 5.2 #16

Express the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\cos(x_i)}{x_i} \Delta x$ on $[\pi, 2\pi]$

as a definite integral

Solution

the summation becomes an integral symbol

the expression involving x_i becomes an expression involving x

the Δx becomes a dx

$$\int_{x=\pi}^{x=2\pi} \frac{\cos(x)}{x}\, dx$$

the left and right endpoints of the interval become the lower and upper endpoints on the integration symbol

Example 5.2 #26 Express the integral

$$\int_1^{10} x - 4\ln(x)\,dx$$

Solution

the integral symbol becomes a summation symbol with limit in front and lower and upper endpoints $i=1$, $i=n$

the expression involving x becomes an expression involving x_i

the dx becomes Δx

the lower and upper endpoints on the integration symbol become the left and right endpoints of the interval

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} x_i - 4\ln(x_i)\,\Delta x$$

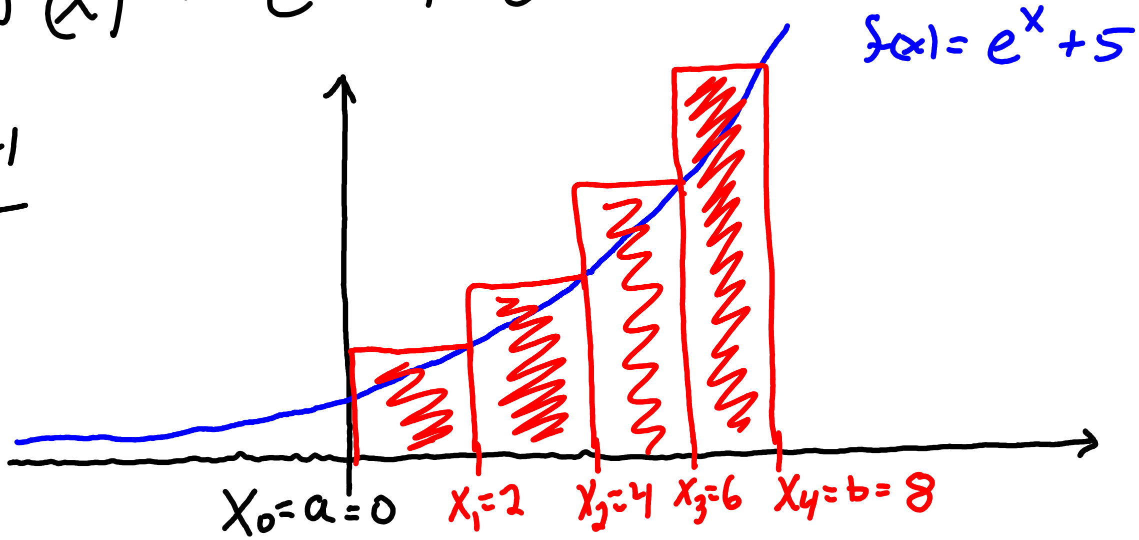
on interval $[1, 10]$

Example Riemann Sum Example

for $f(x) = e^x + 5$ on interval $[0, 8]$

Find R_4

Solution



$$R_4 = [f(x_1) + f(x_2) + f(x_3) + f(x_4)]\,\Delta x$$

$$= [f(2) + f(4) + f(6) + f(8)] \cdot 2$$

$$= [(e^2+5) + (e^4+5) + (e^6+5) + (e^8+5)] \cdot 2$$

$$= [e^2 + e^4 + e^6 + e^8 + 20] \cdot 2$$

$$= \underbrace{2[e^2 + e^4 + e^6 + e^8] + 40}_{\text{exact expression for } R_4} \approx \underbrace{6932.75}_{\text{decimal approximation for } R_4}$$

end of lecture